JOURNAL OF THE AERONAUTICAL SCIENCES

VOLUME 8 APRIL, 1941 NUMBER 6

Air-Cooled *vs.* Liquid-Cooled Aircraft

JOHN G. LEE

United Aircraft Corporation

SUMMARY

Data are presented on the drag, weight, cooling and fuel consumption of air-cooled and liquid-cooled power-plant installations. Based upon these data the design of three groups of air-cooled and liquid-cooled airplanes is postulated. When these groups are compared upon a basis of equal size and weight, the air-cooled aircraft are found to be superior in the pursuit, bomber and transport types.

INTRODUCTION

THERE ARE MANY reasons for choosing an air-cooled or a liquid-cooled power plant. They may be technical, economic or practical. They may depend upon the equipment of certain factories, or the collective experience of a group of engineers, or the previous equipment of an air force. The present paper is confined to the technical reasons.

The presentation of a technical paper on this subject is made difficult at this time because of the restrictions which military necessity imposes upon the publication of new data. Those engineers who are engaged in military work and who have access to confidential material will agree, it is believed, that such material does not controvert the conclusions presented here.

AERODYNAMICS

The basic drag of a streamline object is first considered. In Fig. 1 the drag coefficient is shown plotted against fineness ratio for a variety of both new and old streamline shapes, ranging in form from nacelles to airship models. This curve serves as a reminder that practically speaking, drag is not caused by frontal area alone, but is largely caused by skin friction. In a perfect fluid, a streamline object has no drag at all. Turning to Fig. 2 the skin-friction drags of the streamline forms shown in Fig. 1 have been plotted. These skin-friction drags are presented as coefficients based on the *frontal* area of the streamline form, and are thus directly comparable with the total drag coefficients shown in Fig. 1. It is clear that when we increase the diameter of a fuselage to take a larger engine, the increased drag is due, not to the increase of frontal area, but to the increase of surface area, other things being equal. The data shown in Figs. 1 and 2 have been used to determine fuselage drags in subsequent calculations—increased, of course, for the effects of roughness, scoops and other improprieties.

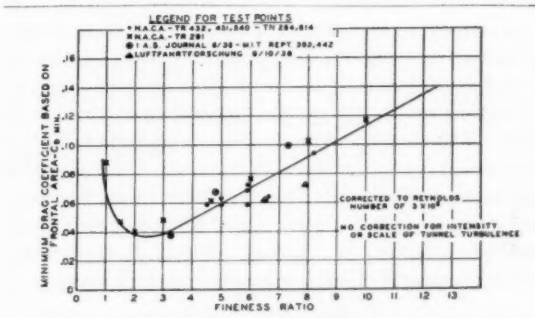

LEGEND FOR TEST POINTS
MINIMUM DRAG COEFFICIENT BASED ON FRONTAL AREA
FINENESS RATIO
CORRECTED TO REYNOLDS NUMBER OF 3 X 10⁶
NO CORRECTION FOR INTENSITY OR SCALE OF TUNNEL TURBULENCE

FIG. 1. Variation of minimum drag coefficient with fuselage fineness ratio.

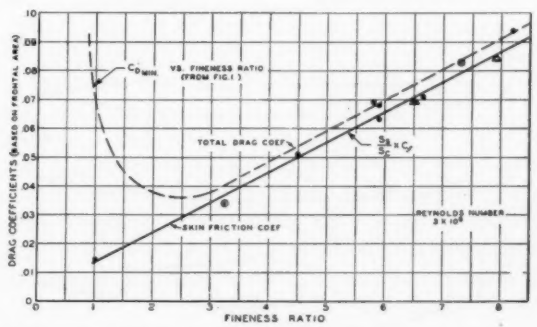

DRAG COEFFICIENTS (BASED ON FRONTAL AREA)
VS. FINENESS RATIO (FROM FIG. 1)
TOTAL DRAG COEF
SKIN FRICTION COEF
REYNOLDS NUMBER 3 X 10⁶
FINENESS RATIO

FIG. 2. Relationship between skin-friction drag and total drag.

Presented at the Airplane Design session, Ninth Annual Meeting, I.Ae.S., New York, January 31, 1941.

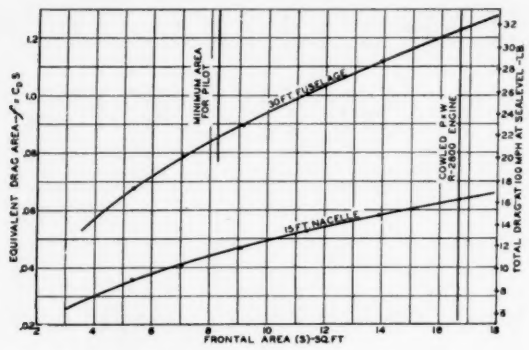

EQUIVALENT DRAG AREA-$f = C_D S$
TOTAL DRAG AT 100 MPH AT SEALEVEL-LBS
FRONTAL AREA (S)-SQ FT
MINIMUM AREA FOR PILOT
30 FT FUSELAGE
15 FT NACELLE
COWLED P & W R-2800 ENGINE

FIG. 3. Drag of typical nacelle and fuselage.

It is interesting to pursue the subject of frontal area and fineness ratio a little further. From Figs. 1 or 2 it can be demonstrated that the total drag of a fuselage 4.5 ft. in diameter and 27 ft. long (Fineness Ratio 6) is actually less than that of a fuselage 4.0 ft. in diameter and 32 ft. long (Fineness Ratio 8), even though the latter has 20 per cent less frontal area. The short, fat fuselage is not as bad as one would like to think. Of course, if the length of the fuselage or nacelle is already determined by other factors, increasing its diameter does increase its drag. This may be seen for two practical cases in Fig. 3. Here the drag, expressed as the familiar f factor, is plotted against cross-sectional area for a nacelle 15 ft. long and a fuselage 30 ft. long. The two vertical lines represent two extremes of area, that at the left is the minimum fuselage cross-section which will house a "jockey" pilot (ellipse 30 in. × 48 in.), and that at the right is the size which will accommodate a 2000 hp. Pratt & Whitney 2800 air-cooled pursuit engine (circle 55 in. diam.). The cross-section required to accommodate a practical liquid-cooled engine, in spite of our best efforts, lies much nearer the air-cooled engine than the "jockey" pilot. Even if we imagine a liquid-cooled engine so compact as to fit, with all its accessories, inside the cross-section required by a "jockey" pilot, the drag of the bare fuselage housing such an engine would still be at least 70 per cent of the drag required for the present Pratt & Whitney 2800 air-cooled engine.

In the case of the drag of an engine nacelle, the situation is complicated by the presence of the wing behind it. In Fig. 4 some unpublished nacelle tests are presented which were run in the $7\frac{1}{2}$ ft. tunnel at M.I.T. on a wing having a 36 in. chord and 60 in. span. The tests were confined to low lift conditions, hence the low aspect ratio is not important. In Fig. 4 the minimum drag and minimum drag coefficient of each nacelle is tabulated, together with a sketch of the nacelle. Unfortunately, the drags of the extension shaft models may be somewhat in error, since they represent the small difference between the measurements of two relatively large quantities. The drags of the other models should be reasonably accurate. In Fig. 5 the

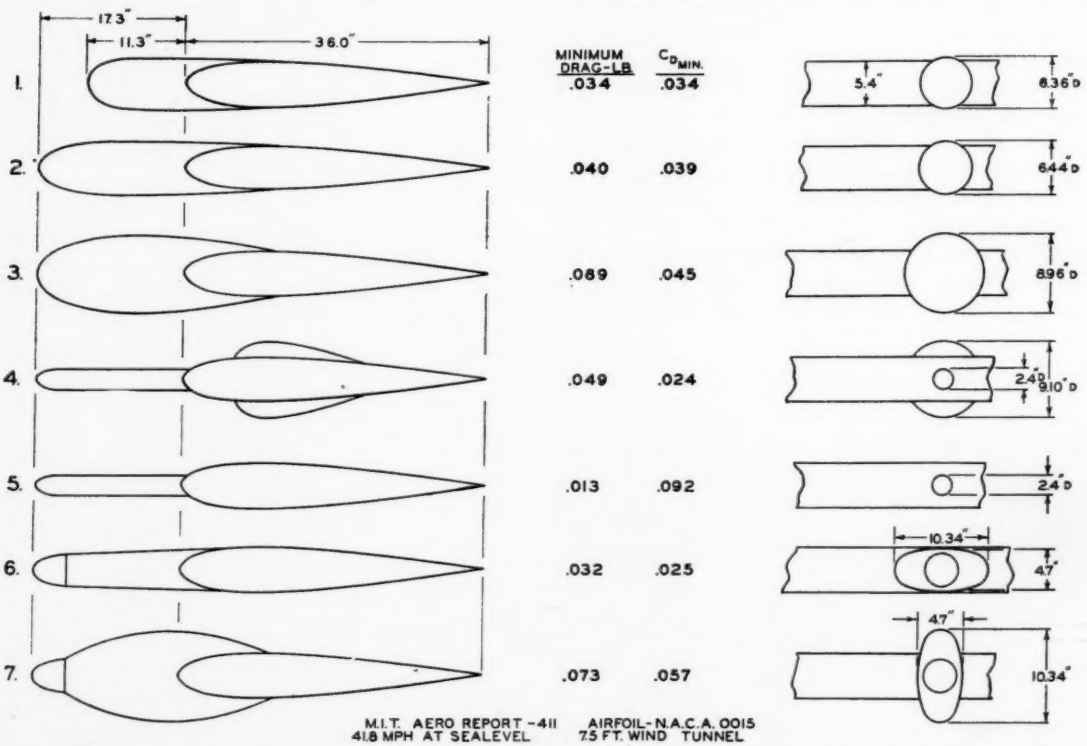


FIG. 4. Nacelle drag and drag coefficients. (No cooling airflow—approximate Reynolds Number 3×10^6.)

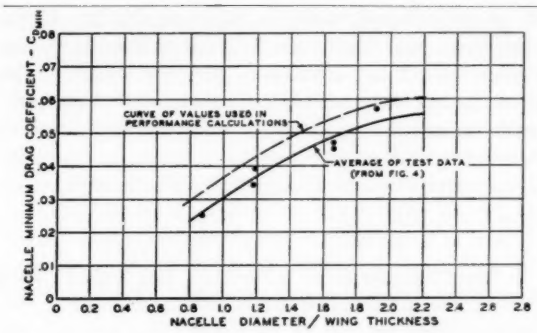


FIG. 5. Variation of nacelle drag coefficient with wing thickness.

minimum drag coefficients of the nacelles in Fig. 4 have been plotted against the ratio of nacelle diameter to wing thickness. A reasonable order prevails. The smaller the nacelle relative to the wing, the lower the drag coefficient. In subsequent calculations the more conservative values given by the dotted line have been used.

A word about the "submerged" engines may be in order at this point. During the past few years wings have been getting thinner for planes of a given gross weight, and it now seems almost impossible to design an engine which may be wholly housed within a wing and which will have sufficient power to be attractive. If the power of these "flat" engines is not really large, higher performance can be obtained in an airplane using larger engines of orthodox type. The tests in Fig. 4 show, however, that considerable improvement can be obtained by mounting "flat" engines horizontally in front of the wing, or by housing radial engines partially within the wing.

Summarizing the material presented so far, it is apparent that increasing the diameter of a fuselage or nacelle of given length results in an increase in drag, due principally to the increased skin friction area and to the interference effects upon the wing. This increase in drag is very much smaller than the increase in area, but is sufficient to count in favor of a compact liquid-cooled engine.

COOLING

Meredith* and others have shown that if air is admitted to a passage at high speed, expanded through a diffuser to a condition of low velocity and high pressure, and then heated, the heat energy imparted to it may be recovered as thrust, if the air is discharged rearwardly through a nozzle or gill. This is the familiar "ducted" radiator of the liquid-cooled engine. It is also the air-cooled engine itself. It makes no difference what the source of heat is, whether it be radiator, engine, or open flame, the same principles apply. To quote Meredith specifically—"By correct design of low velocity cooling systems, in which the surface (whether in the form of a honeycomb radiator or of the fins on the cylinder heads and barrels) is exposed in an internal duct, the power expended on cooling does not increase with the speed of flight but . . . on the contrary, it should diminish to the vanishing point at a practicable speed, beyond which the cooling system contributes to the propulsion."

The only difference between the radiator and the air-cooled engine is that the radiator can be fitted with duct work without modification of the remainder of the cowl, and hence engineers found this a relatively simple change to make. It was made first. Meredith's principles have been recently applied to the air-cooled engine, and the problems appear not to be particularly difficult. As the speed of airplanes increases, the effectiveness of this method of energy recovery improves. The important point is not whether the air-cooled or the liquid-cooled engine has the lower cooling drag, but that the cooling drag of both is being rapidly reduced, and may ultimately become zero. This will occur when the heat energy recovered equals the duct loss.

Any system of heat energy recovery from the engine requires heat transfer from the engine, ultimately, to the cooling air. Heat transfer, to be efficient, requires that the heat be available at the highest possible temperature. Hence, the high temperature air-cooled cylinders are intrinsically more effective for heat energy recovery than the lower temperature coolant radiators. It should be noted also that only a minimum amount of cooling air should be wasted. This is where the "ducted" radiator has had an advantage in the past. The baffles of the air-cooled engine have not been good enough. Improvement in the internal cowling and baffling of air-cooled engines is under way.

In connection with the general problem of heat energy recovery, the smoke-flow photographs taken by Mr. Roger Griswold, II, and shown in Fig. 6, may be of interest. The flow shown in these pictures is two-dimensional outside of the model but three-dimensional inside. Two conditions of flow around a typical engine nacelle are shown; that on the bottom represents the condition where no air is passing through the engine, and the flow around the exit gills produces a considerable suction. That on the top shows the same arrangement where air is admitted to the nacelle under pressure through the back wall of the tunnel and is allowed to flow out through the exit gills. Thus, the bottom flow represents the limiting condition where the air stream is giving up energy to do work in sucking air through an engine, while the top flow represents the condition where energy is supplied to the cooling air by means of heat or by a blower, or any other device, so that the air issuing from the exit gill has substantially the same velocity as the main stream. As might be expected, the bottom flow results in a con-

* Meredith, F. W., *Cooling of Aircraft Engines* R. & M. 1683, August, 1935.

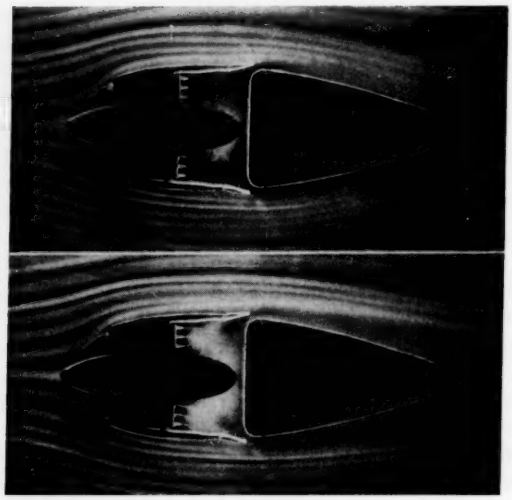


FIG. 6. Smoke flow around two-dimensional models showing disturbance caused (bottom view) by suction at the cowl flaps.

siderable disturbance, while the top flow is smooth. The condition on the inside of the cowl differs in the two pictures; that on the bottom contains smoke, that on the top, air. This fact has no significance in the comparison. Photographs of this sort cannot be used for any quantitative calculation, but they do indicate that as we learn to handle the flow of cooling air through an air-cooled engine, the power required to accomplish cooling will ultimately be reduced to a very small figure indeed, with no harmful effect on the external drag of the remainder of the airplane.

Another of Mr. Griswold's smoke-flow photographs in Fig. 7 shows the air entering the model more clearly

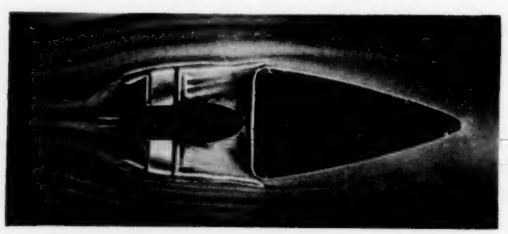


FIG. 7. Smoke flow entering a model.

than in Fig. 6. Notice in particular how the streams approaching the model become wider, which indicates that they are slowing down. From this photograph it appears that the velocity of flow has been reduced about 50 per cent before it enters the N.A.C.A. cowl. Thus, we have obtained a virtually perfect diffuser, without providing either space or weight. This model also was fitted with internal smoke jets to define the internal flow; it is not believed that these jets are of sufficient power to affect the external flow.

Throughout the remainder of this paper the cooling power for both air-cooled and liquid-cooled engines has been arbitrarily taken as 3 per cent of the engine take-off rating. This value represents a conservative average from Pratt & Whitney two-row radials. The latter engines have values of approximately 2 per cent. Furthermore, this assumption does not make use of Meredith's heat energy recovery, which, as has been shown, will reduce the cooling drag of both types.

FUEL CONSUMPTION

In any discussion of air-cooled and liquid-cooled engines, the question of the relative fuel consumption always comes up. It is recognized by all that the fuel consumption under full throttle conditions of the air-cooled engine is materially poorer than for the liquid-cooled engine. However, no engine, whether military or commercial, flies at wide open throttle for more than a very small fraction of its total time. Even the pursuit plane must spend the majority of its time looking for its enemy, or getting from its base to the probable scene of operations. The significant fuel consumption, then, is the cruising fuel consumption.

Much conflicting data has been presented on this subject. Since the compression ratio, supercharging, mechanical efficiency, and several other factors influence the fuel consumption, it is almost impossible to compare a group of liquid-cooled engines with a group of air-cooled engines and obtain any generalized result. In an attempt to eliminate these extraneous factors, Fig. 8 may be of interest. Fig. 8 shows the indicated

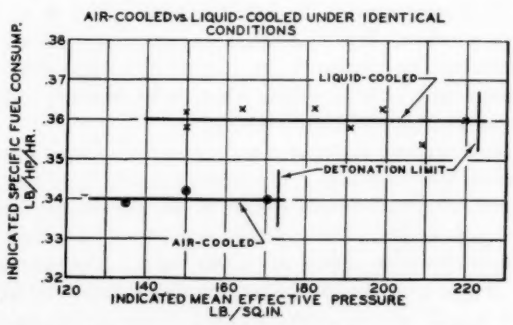


FIG. 8. Best economy fuel consumption tests. (Pratt & Whitney 1830 single cylinder engine.)

specific fuel consumption of two single-cylinder engines run under identical conditions; in fact, they are really the same engine. In the lower curve the engine is air-cooled. In the upper curve the fins were machined off the cylinder, a liquid-cooled jacket substituted, and the same identical cylinder retested as a liquid-cooled engine. The tests were run at the same speed, valve setting, spark advance, and all other controllable conditions.

It is apparent that the air-cooled cylinder is definitely more economical than the liquid-cooled, irrespective of the mean effective pressure. Note that the fuel consumptions are based on indicated horsepower, that is, brake horsepower plus friction horse-

power. If the fuel consumptions are divided by the mechanical efficiency of the engine, which may be of the order of 85 per cent, specific fuel consumptions will be obtained close to the usually quoted values. It may also be noted that these tests were run two years ago, and considerable improvement has since been made in the detonation limit of air-cooled engines.

It may be appropriate to remark that the United Aircraft Corporation has developed liquid-cooled experimental engines continuously over a considerable period of years. Consequently, it is justifiable to state that the data shown in Fig. 8, plus fuel consumption tests on a number of complete engines, both by United Aircraft and by others, lead inevitably to the conclusion that, under cruising conditions, the air-cooled engine is definitely more economical than the liquid-cooled. However, in making this study, the same fuel consumption figures have been used for both types, which is probably justified, since it will permit a certain amount of wide open running, at which time the liquid-cooled engine has the better economy.

Weights

In studying the comparative weights of air-cooled and liquid-cooled installations, the actual weights of a considerable group of engines and their installations were investigated. This resulted in a mass of data, much of it confidential, but in all events too extensive to be presented in this paper. In addition, many of the units could not be assigned permanent weight values. The propeller size and weight, for instance, depends upon the speed and type of the machine, the engine gear ratio, etc. Consequently, the weight data has been summarized in Table 1, giving the limits within which the unit weights were varied throughout this study.

Whenever possible, actual weight values were used. It will be seen that some of the weight values varied over rather wide limits. The liquid-cooled engine weights, for instance, have a range of about 15 per cent. This is because some engines included two-stage gear-driven superchargers while others relied upon turbo installations. In all cases, the individual weights were picked with due consideration and with every effort to make them strictly comparable.

As an off-hand comparison between the weight of air-cooled and liquid-cooled installations, it is interesting to compare the average values from Table 1. All figures are expressed in pounds per take-off horsepower rating. The average air-cooled engine weight is 1.31 lb./hp. and its installation weight is 0.63 lb./hp. making an average installed weight of 1.94 lb./hp. The corresponding figures for the liquid-cooled engine are 1.18 lb./hp. for the engine dry, and 0.91 lb./hp. for the installation making an average installed weight of 2.09. This is some 7.5 per cent heavier than the air-cooled installation.

Table 1

Comparison of Engine Installation Weights

	Air-Cooled	Liquid-Cooled
Engine weight		
Engine dry weight—lb./hp.[a]	1.26 –1.35	1.10 –1.26
Engine power—hp./cu.in.[a]	0.66 –0.75	0.70 –0.81
Power plant installation weight (lb./hp.)		
Propellers and controls	0.302–0.320	0.302–0.320
Starting system (less batteries)	0.020–0.045	0.020–0.045
Engine mount	0.045–0.080	0.045–0.080
Cowling	0.063–0.100	0.063–0.100
Oil system (inc. tanks)	0.040–0.060	0.040–0.060
Fuel system (less tanks)	0.020–0.040	0.020–0.040
Exhaust system	0.027–0.040	0.027–0.040
Radiators and coolant	... ...	0.275–0.300
Miscellaneous[b]	0.015–0.040	0.015–0.040
Total normal inst. wt. lb./hp.	0.532–0.725	0.807–1.025
Additional power plant installation items		
Intercoolers and ducts	0.090–0.095	0.090–0.095
Turbo-supercharger install.	0.109–0.132	0.109–0.132
Gearing, shafting—twin prop.[c]	0.344–0.354	0.344–0.354
Gearing, shafting—single prop.[c]	0.072–0.086	0.072–0.086

[a] These values used when manufacturer's ratings were not available. Engine power is take-off rating.

[b] Includes diaphragms, carburetor air intake ducts, power plant controls, etc.

[c] Twin-propeller installation is for tractor airplane with right angle drives. Single-propeller installation is for pusher or tractor airplane with extension shaft.

Comparison with Present Practice

The evidence presented thus far indicates that the cooling power and the fuel consumption of the air-cooled and liquid-cooled engines may be taken as substantially equal, but that the liquid-cooled installation has a certain advantage in drag and the air-cooled has a corresponding advantage in weight. It is not very satisfactory to try to evaluate the one advantage in terms of the other. For example, weight saved in the engine installation permits a weight saving in the structure; both savings permit a smaller airplane, which has less drag, and hence requires less fuel; this in turn permits another weight saving, and so on. The only practical method is to postulate a group of comparable airplanes, each designed from the ground up, in accordance with the proper area, the proper fuel load, etc. This is a tedious method, involving a considerable amount of "cut-and-try," but it is the only method which includes all the interacting factors, and is the method used in this study.

Before going on to this generalization, it is advisable to check the values of weight, drag, etc., which have been presented, against actual present-day performance. This has been done for the pursuit type airplane. In Table 2 the drag coefficients of five conventional modern air-cooled pursuit planes and five conventional modern liquid-cooled pursuit planes have been tabulated. These figures have been obtained from flight

TABLE 2

Drag Coefficients of Conventional Modern Pursuit Airplanes

Airplane	Drag Coef.	Virtual Drag Coef.[a]
Air-cooled pursuits		
A	0.0196	0.0213
B	0.0230	0.0258
C	0.0216	0.0239
D	0.0224	0.0210
E	0.0173	0.0164
Average Air-Cooled Values		0.0217
Liquid-cooled pursuits		
F	0.0173	0.0176
G	0.0167	0.0178
H	0.0198	0.0211
J	0.0224	0.0202
K	0.0224	0.0220
Average Liquid-Cooled Values		0.0197

[a] "Virtual drag coefficient" represents the normal drag coefficient adjusted to a common value of wing-loading of 27.0 lb./sq.ft.

test, and are "all-up" figures, including the drag of full military equipment and the thrust of exhaust jets, when present. The engine power was known, and only the propeller efficiency required estimation. For obvious military reasons it is impossible to identify these airplanes, or to quote the particular speeds and powers involved. Some are foreign and some domestic. It is believed that the figures are reliable.

The drag figures are presented as coefficients based upon wing area, in the manner of recent N.A.C.A. reports. However, since the pursuits all had different wing loading, these coefficients are not strictly comparable. Airplanes with high wing-loading tend to show abnormally high-drag coefficients. Accordingly, the speeds of all airplanes were corrected to a common wing-loading of 27.0 lb./sq.ft., the average of the group, and the drag coefficients were recalculated, based upon the new wing area and speed. These are the values quoted in Table 2 as "Virtual Drag Coefficients," and are strictly comparable, so that they can be averaged. In making the speed corrections, only the wing chord was changed.

Starting with the average drag coefficients in Table 2, an air-cooled and a liquid-cooled pursuit plane of conventional type were calculated, each weighing 7000 lb., and each having a wing loading of 27.0 lb./sq.ft., which is the average loading for the planes in Table 2. It was found that these average drag coefficients could be easily justified using the drag data presented early in this paper. Since the weight was fixed at 7000 lb., the only "open" variable was the power of the engines. This was arrived at by "cut-and-try," making the proper adjustment for fuel weight, installation weight, etc. The propeller efficiency was estimated by the usual methods. The constants used in this comparison are tabulated in Table 3. A study of this table shows that the drag, weight and cooling data presented in this paper appear to be in accord with the results of flight test.

TABLE 3

Weight and Drag Analysis of Hypothetical Pursuit Airplanes

	Air-Cooled	Liquid-Cooled
Engine rated power, b.hp. at 19,000 ft.	1250	1150
Gross weight, lb.	7000	7000
Wing loading, lb./sq.ft.	27.0	27.0
Wing area, sq.ft.[a]	259	259
Tail area, sq.ft. (25 per cent wing area)	65	65
Max. speed at 20,500 ft., m.p.h.	363	364
Fixed weight, lb.[b]	1500	1500
Structural weight, lb. (8.0 lb./sq. ft.)[c]	2070	2070
Power plant installation weight, lb.[d]	810	1090
Fuel and oil weight, lb. (0.70 lb./hp.)[e]	870	810
Engine bare weight, dry, lb.[f]	1750	1530
Gross weight, lb.	7000	7000
Power loading, lb./hp.	5.6	6.1
Average drag coefficient, C_D	0.0217	0.0197
Equivalent drag area, $f = C_D S$, sq.ft.	5.62	5.10
Max. frontal area includ. cockpit windshield, sq.ft.	16.3	12.8
	Equivalent Drag Area, f	
Item		
Wing ($C_D = 0.0090$) } Including roughness,	2.330	2.330
Tail ($C_D = 0.0095$) } gaps, etc.	0.618	0.618
Fuselage ($C_D = 0.095$)	1.549	1.216
Miscellaneous	0.760	0.760
Intercooler	0.175	..
Oil cooler	0.025	0.025
Cooling at 3 per cent	0.164	0.149
Total drag area, f	5.62	5.10

[a] Aspect ratio 5.70.

[b] Fixed weight includes pilot, armor, 4 machine guns, ammunition, controls, radio, furnishings and instruments.

[c] Structural weight includes wings, tail, fuselage, landing gear and control system.

[d] Power plant installation weight 0.65 lb./hp. for air-cooled and 0.95 lb./hp. for liquid-cooled. Includes engine mount, intercoolers, cowling, exhaust system, engine accessories, ducts, bullet-proof tankage, fuel and oil systems and coolant and radiators on liquid-cooled type.

[e] Fuel and oil based on 0.040 lb. oil consumption and 0.50 lb./hp.hr. fuel consumption at 50% rated power for 2.6 hr.

[f] Engine bare weights based on typical two-stage air-cooled and two-speed liquid-cooled engines of 1.40 lb./hp. and 1.33 lb./hp., respectively.

The most interesting point in Table 3 is the resulting performance of the two airplanes. The speed of the air-cooled pursuit comes out 363 m.p.h.; that of the liquid-cooled, 364 mph. Both airplanes are the same size and weight. Both have drags which are the average of current practice. However, the air-cooled engine comes out with 1250 hp. and the liquid-cooled with 1150 hp., both at an altitude of 20,500 ft. The

engine specific weights are taken from the nearest available pursuit engines. Fuel is provided to give each plane the same endurance.

One hesitates to argue from the specific case to the general, but here is an instance where the speed of the air-cooled and the liquid-cooled pursuits are substantially equal, both have the same wing loading, but the air-cooled has a materially lower power-loading, (5.6 lb./hp. air-cooled; 6.1 lb./hp. liquid-cooled). Hence the take-off, climb and ceiling will all be materially in favor of the air-cooled. In short, the air-cooled is superior.

Analysis of Future Pursuit Airplanes

Using the data presented in the first part of this paper, certain groups of airplanes have been postulated and performance of air-cooled and liquid-cooled engines compared in each group. Certain of the engines are now in production; others are modifications of present engines; still others are only on paper.

The first group of airplanes to be considered is the pursuit type, such as that shown in Fig. 9. This

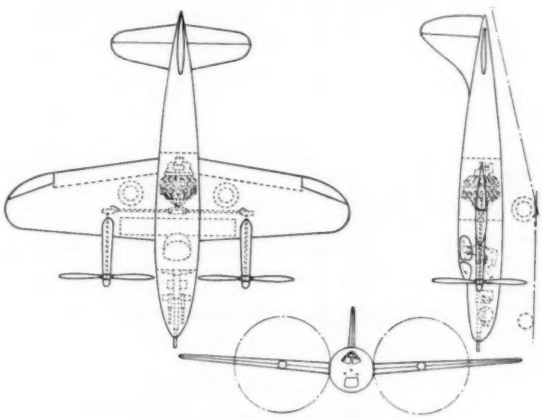


Fig. 9. Pursuit airplane.

somewhat unorthodox arrangement was chosen because it permits the smallest possible fuselage, considering the size requirements of the pilot and the engine; it permits the installation of cannon or other heavy armament directly in front of the pilot; it eliminates the violent torque reactions because of its opposite rotating propellers; and finally, it permits egress from the airplane with a parachute. Admittedly, this type of pursuit plane involves the weight and complication of shaft drives and gearing, and it is not particularly suited to laminar flow wings. One cannot have everything. Parallel calculations were made on a group of strictly orthodox pursuit airplanes, and on another group which were of the single propeller pusher type. The conclusions reached from the study of these other types were identical to the conclusions reached in the study of the type in Fig. 9. Hence, it is fair to say that the particular arrangement does not favor either air-cooled or liquid-cooled engines.

Table 4

Weight and Drag Constants Used in Figs. 10 to 17

	Pursuit		Bomber		Flying Boat	
	Air.	*Liq.*	*Air.*	*Liq.*	*Air.*	*Liq.*
Engine bare lb./hp.[a]	1.35	1.26	1.32	1.24	1.26	1.10
Pwr. plnt. inst. lb./hp.	.671	.946	.710	.990	.590	.880
Fuel & oil lb./hp.hr.	0.470[b]		0.480[c]		0.480[d]	
Fixed weight—lb.[e]	1440		6550		5780[f]	
Non-struct. wt.—lb.	(Summation of above items)					
Ratio struct./non-struct.	36.73		53.43		76.2[g]	
Gross weight—lb.	(Summation of struct. and non-struct. wt.)					
Wing loading—lb./sq.ft.	40		40		40	
Aspect ratio	5.50		9.50		10.0	
Ratio tail/wing area	28.0		40.0		30.0	
Ratio length—span	94.0[h]		75.0		70.0	
Wing C_D	0.0062		0.0063		0.0060	
Tail C_D	0.0062		0.0060		0.0053	
Fuselage or hull C_D[i]	.071–	.137	.077–	.200	.083–	.110
Nacelle C_D[i]		.09[j]	.028–	.078	.028–	.057

[a] Used when manufacturer's specifications were not available—based on take-off power.

[b] Based on cruising endurance of two hours.

[c] Based on 3000 mile cruising range at 50 per cent take-off power.

[d] Based on 3500 mile cruising range against continuous 30 m.p.h. headwind at 50 per cent take-off power.

[e] Includes crew with chutes, radio, instruments, cockpit accommodations, armament, food, water, etc.

[f] Based on requirements of 40 passengers.

[g] Average value.

[h] Value for twin-propeller tractor with shaft drives.

[i] Value for C_D based on frontal area.

[j] C_D for extension shafts.

In Table 4 are listed the weight and drag constants used in the study of not only the pursuit airplane but also of the bombing and flying-boat types which follow. Every effort has been made in the selection of these various constants to make the comparison between air-cooled and liquid-cooled strictly fair. In particular, the size of the fuselage was adjusted to the size of the engine, the wing area was adjusted to the weight, etc. The necessity for making these adjustments is paramount. The airplane must be designed to fit the engine. The procedure of putting a fat air-cooled engine in a slim liquid-cooled fuselage, or vice versa, and letting the wing-loading do what it may, gives a false comparison. Such installations may be a useful military expedient when haste is essential, but air-cooled and liquid-cooled engines cannot properly be compared on that basis. The airplane must fit the engine.

Turning now to Fig. 10, the top speed of a group of pursuit airplanes of the type shown in Fig. 9 has been plotted against the brake horsepower. The extremely high speeds shown are the result of a uniform assumption that the present take-off power rating will be

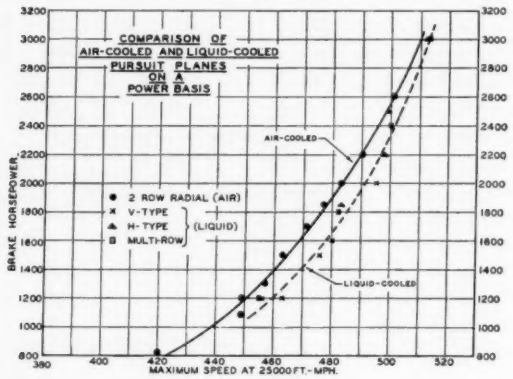


FIG. 10. Pursuit planes compared on a power basis.

available in the future at 25,000 ft., that compressibility burble can be postponed, and that a 40 lb./sq.ft. wing loading can be ultimately employed. It appears that the liquid-cooled planes are roughly 10 m.p.h. faster than those fitted with air-cooled radials. Looked at another way, the air-cooled planes would require some 250 additional horsepower to attain the same high speed as the equivalent liquid-cooled planes. The really important fact is that for planes of equally clean design, much greater gains may be made by increasing the power than by changing the type of cooling. If a 2000 hp. air-cooled engine is available, liquid-cooled engines of 1200 or 1500 hp. simply cannot compete with it.

We are accustomed to comparing airplanes on the basis of equal power and pay little attention to the fact that the liquid-cooled airplane, being heavier, is also the larger. To build the larger airplane costs more money and takes more time. We are primarily concerned not with the power of the airplane but with its performance, its cost, and the difficulty of building it. We must include in that performance not only its high speed, but its climbing, landing, and maneuvering ability. To give equal maneuvering ability, that is, to be able to maneuver within the same space, requires that the air-cooled and liquid-cooled airplanes have the same wing loading. Landing requirements also demand equal wing loading. This results in the liquid-cooled being the larger and heavier airplane. When we consider airplanes on the basis of equal gross weight and wing loading, much of this difficulty disappears. The air-cooled and liquid-cooled types are then on a par not only as far as construction is concerned, but also in regard to landing and maneuverability characteristics.

Turning now to Fig. 11, the high-speed of the same group of pursuit airplanes has been plotted against gross weight instead of brake horsepower. It is apparent that the high speed of airplanes of equal gross

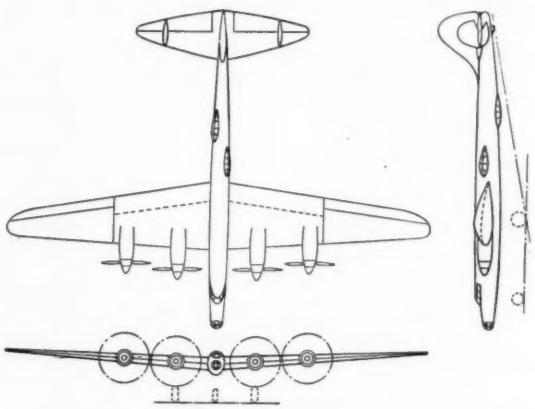


FIG. 12. Typical bomber.

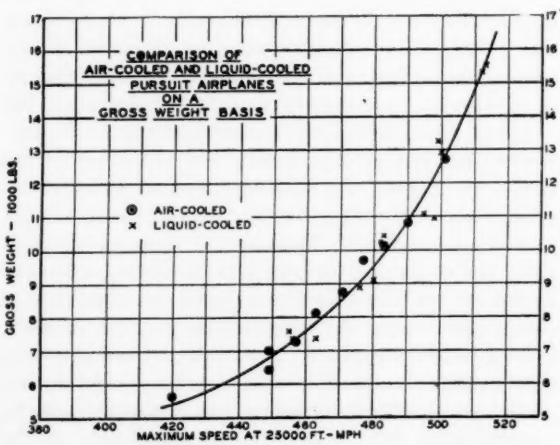


FIG. 11. Pursuit planes compared on a weight basis.

weight is substantially the same. The superiority of liquid-cooled compactness has been balanced against the superiority of air-cooled lightness, and we find that for airplanes of the same gross weight and the same handling characteristics, it makes no difference which you use. The air-cooled engines merely have a slightly larger amount of power, and include the additional fuel required by that larger power. This results in a definitely lower power-loading for the air-cooled type, which means that the take-off, climb, and ceiling are all definitely superior to those of the liquid-cooled type. Specifically, comparing two pursuit planes of 9800 lb. gross weight, the air-cooled would have 2000 hp. and the liquid-cooled 1750 hp. Both would have the same top speed, but the take-off of the air-cooled would be approximately 15 per cent shorter, the rate of climb 20 per cent more, and the ceiling 3 per cent higher than the liquid-cooled. In short, the air-cooled pursuit is superior.

Analysis of Future Bombing Airplanes

In Fig. 12 is shown a 3-view drawing of a typical bombing airplane, of the 4-engine type. Similar studies were made of twin-engine bombers carrying a slightly reduced crew but otherwise identical. The same conclusions were reached with both types, so only the 4-engine type is discussed here. It should be noted that these bombers are of the high-speed class (350–450 m.p.h.) and carry a relatively small load (2000 lb.) over a 3000-mile range (in still air). Appreciably larger bomb loads could be carried with a slight increase in take-off run, which would undoubtedly be done in war time. The size of the fuselage will accommodate bomb loads up to 10,000 lb. The performance of these bombing planes has not been projected so far into the future as the pursuit planes. The cruising speed has been taken at 15,000 ft., corresponding to 50 per cent of the rated take-off horsepower of the engines. Many of these engines are already in existence, consequently bombers of this general class could be built now.

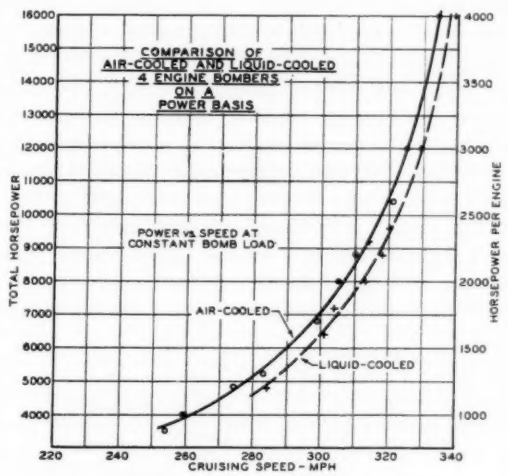


Fig. 13. Bombers compared on a power basis.

In Fig. 13 the cruising speed of the bombers is plotted against their power. As in the case of the pursuit plane, the liquid-cooled planes have a superiority in speed at the same power amounting to approximately 6 m.p.h. at cruising, or, expressed the other way, the air-cooled engines require an increase of roughly 250 hp. per engine to give the same speed.

The same argument holds true with bombers as with pursuit airplanes, namely, that the cost of building the airplane and the size of the facilities required depend primarily upon the size and weight of the airplane, not upon its power. In Fig. 14 the performance of the same group of bombers has been plotted against gross weight. Again it is found impossible to differentiate between the air-cooled and liquid-cooled types. The same speed may be obtained with either, but, as

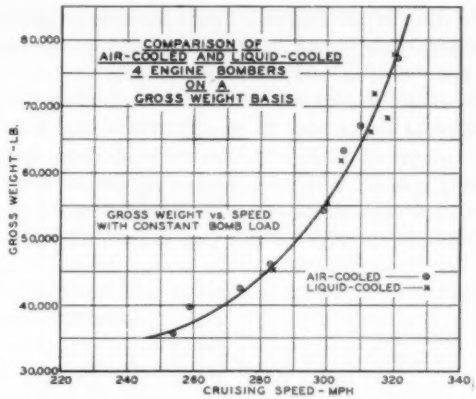


Fig. 14. Bombers compared on a weight basis.

before, the take-off, climb and ceiling are all clearly in favor of the air-cooled type, with its lower power-loading. Again we must conclude that the air-cooled is superior.

Analysis of Commercial Flying Boats

In Fig. 15 is shown a typical commercial trans-oceanic flying boat of generally orthodox arrangement. These boats have been studied on the basis of a 3500-mile cruising range against a continuous 30 m.p.h. headwind at sea level, using 50 per cent of the engine take-off power. Calculations have been made not only of pay load, but of operating costs. In computing this operating cost, only those figures which are directly influenced by the choice of airplane equipment have been included. Items such as weather service, terminal facilities, advertising, and the like have been omitted. However, depreciation, maintenance, repair, servicing, crew's salary and expense, etc., have been included. Extensive use has been made of the data presented by Mentzer & Nourse* in their paper

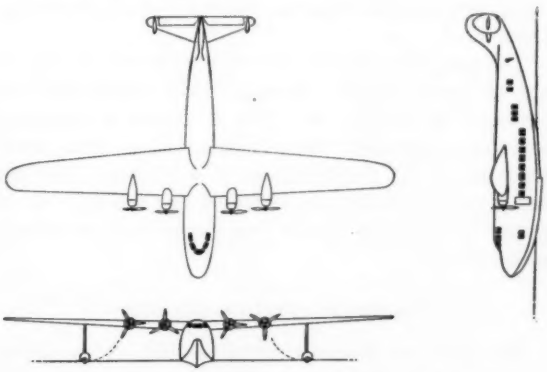


Fig. 15. Typical commercial transoceanic boat.

* *Some Economic Aspects of Transport Airplane Performance*, Journal of Aeronautical Sciences, Vol. 7, No. 6 and 7, April and May, 1940.

before the I.Ae.S. Eighth Annual Meeting in January, 1940, but with proper modification due to the use of overwater aircraft and the longer route.

There are several ways in which the comparison of commercial boats may be made. Since the bombers were compared with equal bomb loads and variable cruising speeds, it will be interesting to compare the boats with variable pay load and equal cruising speeds. Accordingly, in Fig. 16 the pay loads and operating

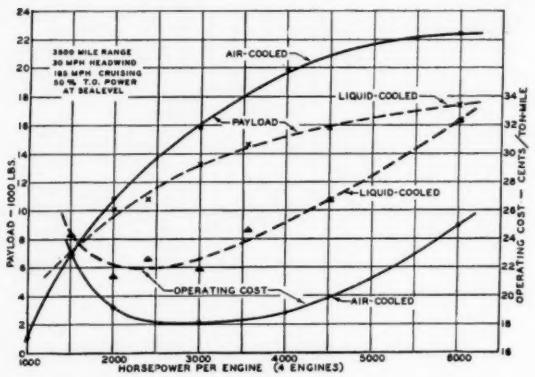


FIG. 16. Comparison of 4-engine air-cooled and liquid-cooled flying boats on a power basis.

costs of a group of flying boats cruising at 185 mph. (still air) have been plotted against engine horsepower. In this case the superiority of the air-cooled engine is apparent at once. On the basis of equal horsepower, the air-cooled installation carries the greater pay load and hence has the lower operating costs, excepting in engines of the 1000–1500 hp. class. In this class the air-cooled has been taken as a single-row radial, which has large frontal area per unit of power. In all other cases the air-cooled engines are of the multi-row type. It is well to recall that equal specific fuel consumptions have been figured for both air-cooled and liquid-cooled types, although the evidence indicates that better cruising fuel consumption can be attained with the air-cooled type.

The same information has been replotted in Fig. 17 against gross weight. Here the same conclusions are apparent as in Fig. 16. The air-cooled is definitely superior. Although these studies have been made for commercial flying boats, it is obvious that the same results would be obtained in bombers or in patrol boats for the same long range and heavy-load requirements.

Discussion and Conclusions

We have not discussed serviceability, vulnerability, or engine cost. These are difficult qualities to handle quantitatively. A very good qualitative case can be made for the superiority of the air-cooled engine as regards serviceability and vulnerability. A bullet through the cooling fins or the baffles, for instance, will not stop an engine. There are definite records of an air-cooled engine continuing to run with a bullet hole through the cylinder and the piston. A bullet through a water jacket brings a plane down. An air-cooled cylinder can be replaced, but a liquid-cooled engine requires a whole new cylinder block. As for cost, those figures which have come to the writer's attention indicate much higher costs for the liquid-cooled engine, as well as for installing the engine, than for the corresponding air-cooled type. Higher cost is an indication of greater difficulty in production, and this is an important military as well as commercial consideration.

In this study it has been clearly brought out that, on the basis of equal weight, equal cleanness, and equal maneuvering characteristics, the air-cooled installation can equal the speed of the liquid-cooled, and surpass it in take-off, climb, and ceiling. When range and weight-carrying ability are the primary consideration, the air-cooled has every advantage. In view of this situation, it is only proper to ask why are not all the engines of the warring nations of the air-cooled type? The answer is not technical; it is practical. The Germans have specialized in liquid-cooled engines ever since the last war. What few air-cooled types they have built have been of American design, built under license. Judging by the fact that the Germans continued to buy American radials for some years, it may be inferred that they never really mastered the air-cooled types. The English have built both types, as did the French. However, the application of Meredith's ducted radiators to the liquid-cooled pursuit planes in advance of the time when his principles were applied to the air-cooled pursuit, gave the liquid-cooled

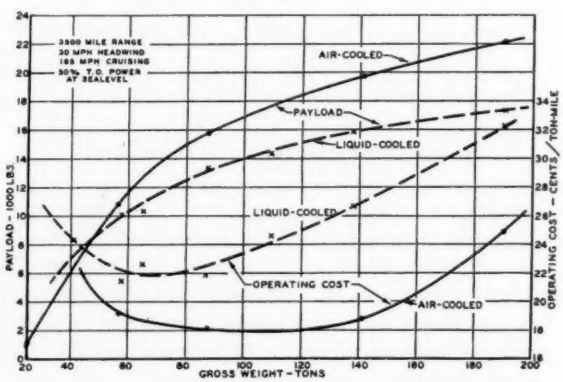


FIG. 17. Comparison of 4-engine air-cooled and liquid-cooled flying boats on a gross weight basis.

type a temporary speed advantage. This advantage came at the time England was going into war production, so it was natural that her pursuit planes would be built around liquid-cooled installations, while her bombers used the air-cooled.

Because the Germans and the English use liquid-

cooled pursuits is not an argument that we should. They made their decision at a time when they lacked important technical knowledge which we now have. They cannot change now. We are in an extremely fortunate position to have in production 2000 hp. air-cooled engines—30 per cent more powerful than anything available abroad. The foreign nations would match them if they could, but they are learning, as some Americans have already learned, that to develop a really satisfactory 2000 hp. liquid-cooled engine is truly a monumental task.

Of course, we must learn from the experience of other nations when that experience applies to us; but we must not copy the mistakes of other nations, nor must we accept their limitations, if they do not apply to us. When we are possessed, as we are today, of technical knowledge which was not available to them at the time when they "froze" their fighting types, we should not hesitate to differ with them. In the light of our present knowledge, comparison of air-cooled and liquid-cooled aircraft upon a basis of equal size, weight and type, indicates that the air-cooled aircraft are superior.

Letter to the Editor

March 14, 1941

Dear Sir:

A graphical solution of flutter problems would be most desirable for the so-called three degree of freedom case, where bending and torsional oscillations of the wing, coupled with angular movement of the control surface, are considered, since in this case the computations become quite involved. For two degrees of freedom, or what is sometimes called the binary case, the work is so very much reduced that it can hardly be called "tremendous" as outlined in Bergen and Arnold's article about this subject.[1] With due consideration of the advantages which a graphical solution offers, it should also be borne in mind that an analytical method is more adapted to take into account extra effects such as structural and friction damping, aerodynamic balance or certain correction factors.

Unfortunately, the results of Bergen and Arnold's article are somewhat in contradiction with results obtained elsewhere. Calling again ω_β the natural circular frequency for angular movement β of the control surface at zero speed and ω_h the corresponding frequency due to translatory movement in the h direction normal to the "fixed" surface, which is mostly due to bending oscillations, they pointed out that "in practical cases the ratio ω_β/ω_h usually exceeds unity," and that values less than unity "are of little practical significance." But even for symmetrical wing oscillations, where the full stiffness of aileron controls comes into play, values ω_β/ω_h encountered are quite often smaller than one. For rudder and elevators they approach zero depending on the amount of "stiffness" with which the pilot might try to hold the controls during flutter, unless a nodal point in the control system, such that one part of the controls vibrates in opposite direction to the rest, is also considered. Values $\omega_\beta/\omega_h < 1$ are actually the most "practical" and important cases for flutter. It appears from Fig. 19 on page 507 that $\omega_\beta/\omega_h = 1$ is some kind of a highly distinguished asymptote such that flutter would occur for all values of center of gravity locations of the movable surface (x_β parameter) if this ratio becomes smaller than one. This is in direct contradiction to the graphs of Theodorsen on page 18,[2] as well as to "practical cases" known. In the "numerical example" for $\omega_\beta/\omega_h = 1.5$ and $x_\beta = 0.0182$ flutter speeds from 192.5 feet/sec. to 349 feet/sec. were evaluated, while again according to Theodorsen's graph II-B no flutter at any speed exists for $(\omega_\beta/\omega_h)^2 = 2.25$ as chosen.

Until these discrepancies are clarified, Bergen and Arnold's results should be used with great caution.

FELIX NAGEL
Airplane Div., St. Louis Plant
Curtiss-Wright Corporation

---

[1] Bergen, W. B., and Arnold, Lee, *Graphical Solution of the Bending-Aileron Case of Flutter*, Journal of the Aeronautical Sciences, Vol. 7, No. 12, page 495, October, 1940.

[2] Theodorsen, Th., and Garrick, I. E., *Mechanism of Flutter*, N.A.C.A. Report No. 685, 1940.

Possibilities of the Two-Stroke Cycle for Small Aircraft Engines

A. R. ROGOWSKI

Massachusetts Institute of Technology

ABSTRACT

In order to reduce the first cost and maintenance of the small airplane, and thus make private flying more universally available, there is need for a simple, reliable, low-cost engine with performance equal to the present small four-stroke unsupercharged power plant. The piston-ported loop-scavenged two-stroke engine is considered for this purpose, using as a basis the results of tests on an experimental cylinder of this type. It is concluded that this engine would be most satisfactory, provided the fuel consumption at high outputs could be reduced by means of relatively inexpensive fuel-injection equipment.

INTRODUCTION

THE PRESENT increase of business which is taking place in the light-plane industry has led many persons to predict that private aircraft production is headed for an expansion similar to that experienced in the early days of the automobile. Others feel that for such a growth to occur, a corresponding reduction must be made in light-plane cost. It is certainly true that many thousands of interested people are prevented from taking up flying for pleasure by the relatively high cost of owning or renting a plane.

At the present time, the power plant accounts for twenty per cent of the cost of a typical small mass-produced motor car. This ratio runs from thirty to forty per cent, however, in the case of typical light planes. In addition to this, it seems probable that the application of production methods to the relatively new field of plane manufacture will effect a greater reduction in plane cost than in engine cost, so that this unfavorable cost ratio may be even greater in the future. In any event a drop in power plant cost will be strongly reflected in the selling price of the complete plane.

An item on modern four-stroke engines responsible directly and indirectly for a sizable fraction of the first cost and subsequent upkeep is the valve gear. The valves, camshaft and cams require expensive materials, which are hard to fabricate and are subject to mechanical or thermal failure in use. The cylinder head construction necessary to accommodate the valve seats is one of the more difficult design problems, and the cost of the cylinder is greatly increased by the extra machining operations involved. The presence of hot exhaust valves may lower the permissible compression ratio, while the valve stems tend to collect objectionable deposits. These troubles have been sufficient to cause many builders of large engines to attempt sleeve valve designs. Such a solution would, of course, be too expensive for light-plane engines.

Presented at the Power Plants session, Ninth Annual Meeting, I.Ae.S., New York, January 30, 1941.

OBJECT

It is not the purpose of this paper to advocate the unreserved adoption of the two-stroke cycle, but merely to encourage a general consideration of this form of power plant for aircraft and, more particularly, to bring to the attention of the light-plane industry the possibilities of the piston-ported, loop-scavenged type of two-stroke engine.

This type of engine possesses many interesting aspects, since valve gear may be completely done away with, the cylinder head may be made without valve ports or inserts, and the entire engine is characterized by the utmost mechanical simplicity. Apparently the neglect of this arrangement in the past was due to the fact that high specific output used to be the prime requisite of all aircraft engines. This led the designers of two-stroke engines to sacrifice cost and mechanical simplicity in an attempt to increase output. The supercharged four-stroke engine was superior in this field.

With the advent of the modern small airplane the picture has changed, and there is now a definite need for an engine with a specific output equal *only* to that of the unsupercharged four-stroke engine, but which, because of its mechanical simplicity, will be cheaper to produce, cheaper to service and equally reliable in flight.

EXPERIMENTAL ENGINE

An experimental single-cylinder engine of this type with a $4^1/_2$ in. bore and 6 in. stroke was built and tested at the Massachusetts Institute of Technology during the last three years. The results of most of this work and a detailed description of the engine appear in N.A.C.A. Technical Notes 674[1] and 756.[2] Fig. 1 shows a cross-section of this engine. The cylinder is enclosed in an outer barrel and is split longitudinally. One section contains the inlet ports and the other the exhaust ports. The crank angle at which the piston uncovers the ports may be varied by sliding one or the other section up or down within the barrel. Suitable inserts are used to vary the shape of the ports as de-

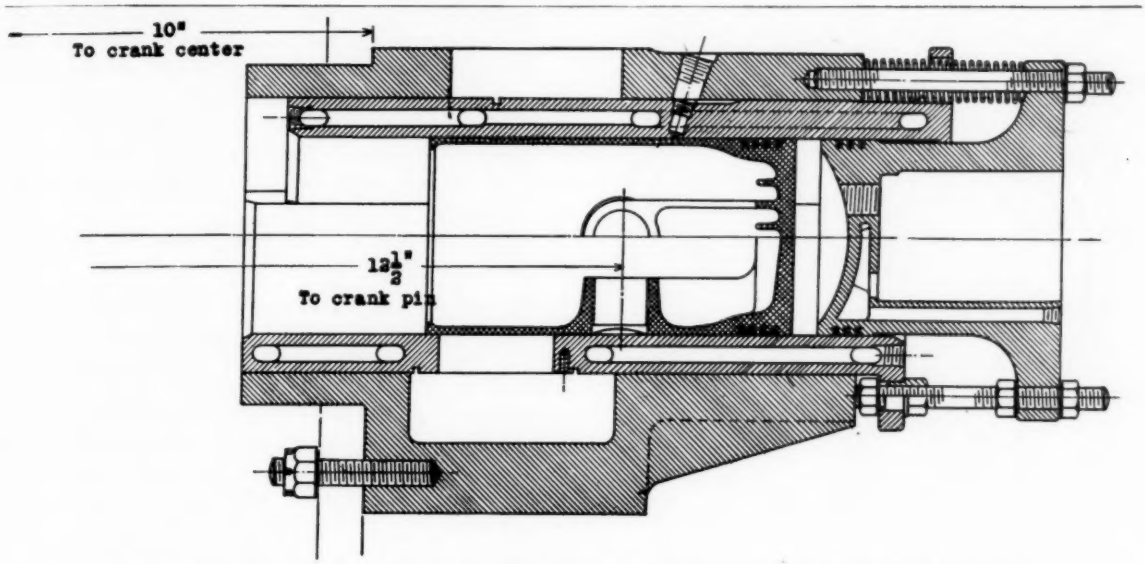


FIG. 1. Cylinder assembly. Sections shown are taken at various angles through center line of cylinder.

sired. The compression ratio may be changed by moving the cylinder head axially in the cylinder.

With this flexible arrangement it was possible to determine experimentally the optimum inlet and exhaust port timing and the best port shape and arrangement. Studies were also made of the best piston-head and cylinder-head shapes.

Fig. 2 shows the split cylinder construction and Fig. 3 gives a horizontal cross-section through the ports. The inlet ports are radial about a point on the inlet side of the cylinder axis. This construction has been described by Curtis[3] and Sulzer,[4] and is for the purpose of causing the scavenging air to rise vertically on the inlet side of the cylinder without the use of a deflector piston. It is to be emphasized here that in the commercial engine the flexible features would be eliminated, leaving a perfectly plain cylinder, perhaps cast in one piece, with a simple piston and rod. The chief moving parts would be the piston, connecting rod, crankshaft and blower.

PERFORMANCE

Output. The performance to be expected from this type of engine using gasoline as a fuel may be estimated from the results of tests made at the Massachusetts Institute of Technology in conjunction with N.A.C.A. Technical Notes 674 and 756. Illuminating gas was used in these tests, but such an analysis will be accurate within a few per cent, which should be sufficient to establish the merits of this form of engine, and it is hoped that experimental verification will soon be available to those interested.

All tests were made under the following conditions.

Compression ratio	7
Spark advance	13° B.T.C.
Port timing:	
Inlet	52°–55° B.B.C.
Exhaust	65° B.B.C.
Fuel-air ratio	0.227 by volume (best power)

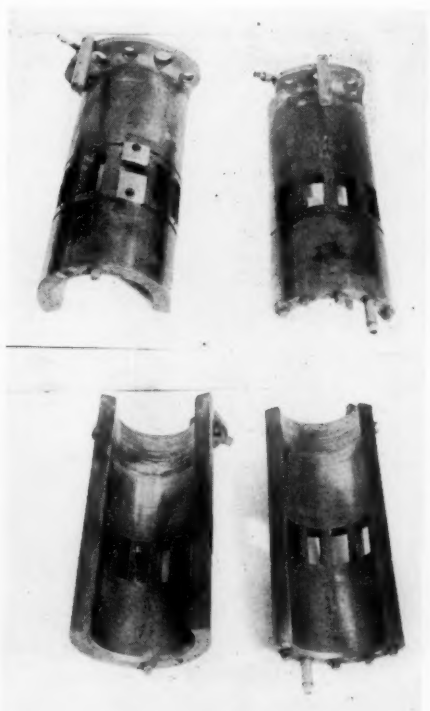


FIG. 2. Cylinder construction.

FIG. 3. Horizontal cross-section through ports.

The following terms used in this report are defined as suggested by Prof. C. F. Taylor in his paper on *An Analysis of the Charging Process in the Two-Stroke Engine.*[5]

Scavenging ratio: The weight of fresh charge delivered through the inlet ports, divided by the weight of fresh charge which will fill the entire cylinder volume at inlet temperature and exhaust pressure, with the piston at bottom center.

Gross brake m.e.p.: The brake m.e.p. developed by an engine with separately driven scavenging blower.

Net Brake m.e.p.: The gross brake m.e.p. minus the scavenging blower m.e.p.

Fig. 4 represents the gross b.m.e.p. obtained at various speeds and scavenging ratios using illuminating gas as fuel. From tests made on various engines in the laboratory, it is known that under the same operating conditions, the ratio of indicated m.e.p. developed with gasoline to that produced with illuminating gas is equal to 1.13. The curves of Fig. 4 may therefore be converted to gross b.m.e.p. with gasoline if the motoring friction of the engine is determined. The gross b.m.e.p. calculated in this manner is shown in Fig. 5.

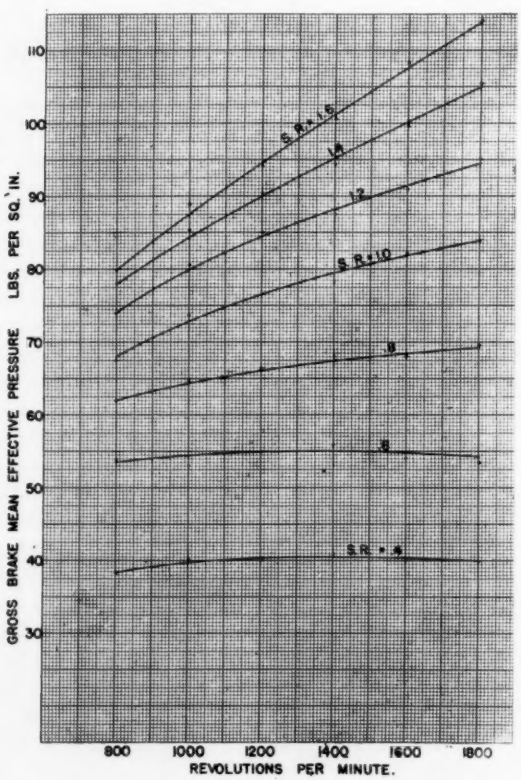


FIG. 4. The effect of engine speed on gross mean effective pressure at various scavenging ratios. Illuminating gas; compression ratio 7; spark advance 13°.

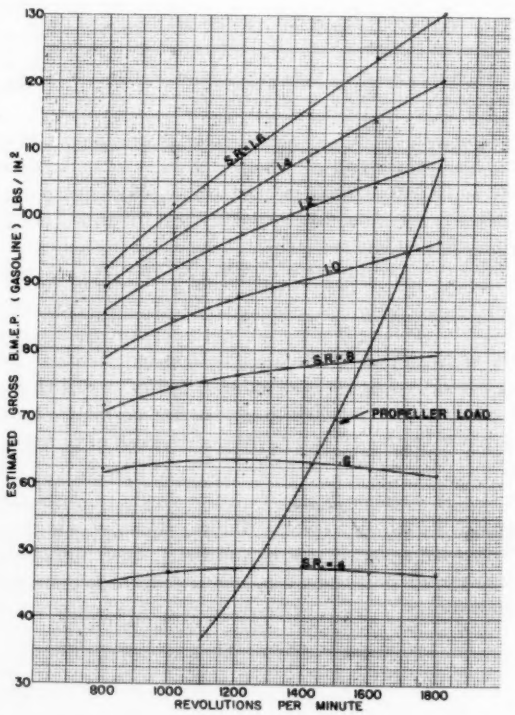


FIG. 5. The effect of engine speed on gross brake mean effective pressure at various scavenging ratios. Estimated performance with gasoline.

The scavenging pressures required by the engine at various speeds and scavenging ratios are given by the experimental curves of Fig. 6. Streamlining the ports was not attempted, and it is probable that some reduction in scavenging pressures could be effected in this way. The required pressure at a given scavenging ratio is seen to vary nearly as the square of the engine

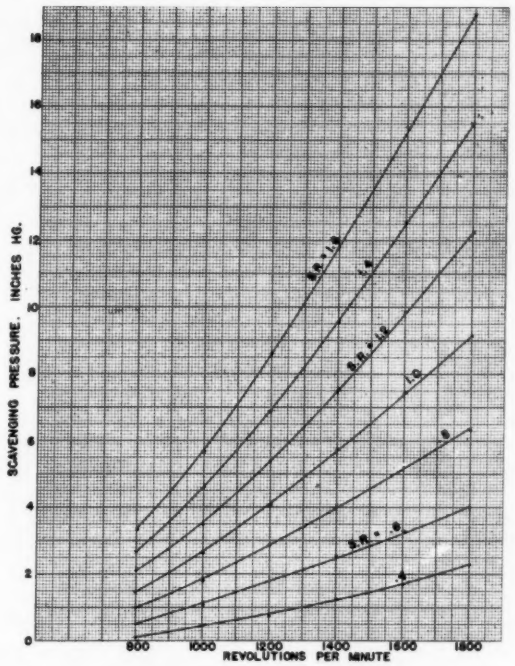


FIG. 6. Variation of required scavenging pressure with engine speed at several scavenging ratios. Experimental values.

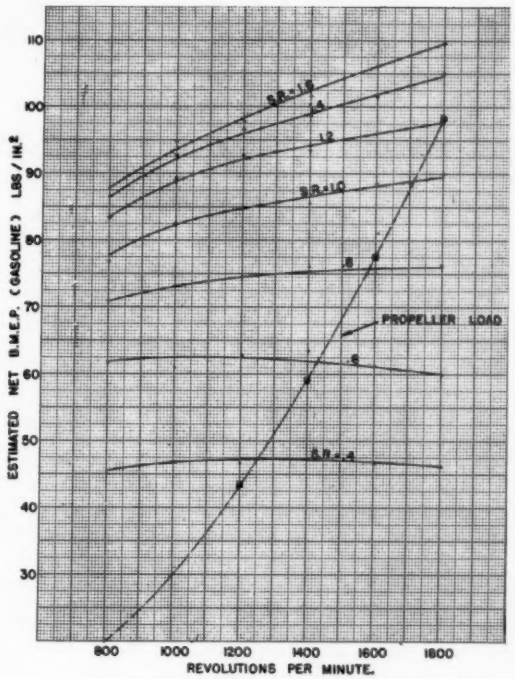


FIG. 7. The effect of engine speed on net brake mean effective pressure at various scavenging ratios.

speed. This agrees with the simple theory which treats the engine as an orifice requiring gas velocities proportional to the engine speed. From the data of Fig. 6 the required scavenging blower m.e.p. was calculated, assuming an adiabatic efficiency of 70 per cent for the blower.

Subtracting these blower m.e.p.'s from the gross b.m.e.p.'s of Fig. 5 results in Fig. 7, the net b.m.e.p.'s which may be expected with gasoline.

A centrifugal scavenging blower geared to the engine will give scavenging pressures approximately proportional to the square of the engine speed. This will therefore keep the engine at nearly constant scavenging ratio throughout the speed range. An inspection of Fig. 7 shows that scavenging ratios above 1.2 are probably not justified, as the increase in net power is small, and the high pressure required would mean a heavier blower and blower drive.

It will be assumed that the blower is designed to deliver a scavenging ratio of 1.2 at an engine speed of 1800 r.p.m. (1800 feet per minute piston speed) at which point the scavenging pressure will be 12.3 in. Hg and the net b.m.e.p. will be 98 pounds per square inch. This is the "full throttle" condition, and the b.m.e.p. is equivalent to 196 pounds per square inch on the four-stroke basis.

A propeller load curve is drawn through the full-throttle point. The angle at which the propeller load crosses the curves of constant scavenging ratio (constant throttle position) in Fig. 7 indicates a high degree of engine speed stability. Fig. 8 represents the blower pressures available at full throttle, and those required on a propeller load curve. The blower pressure ratios required are considerably less than those used in the larger supercharged engines, so that less expensive impeller construction might be used. Impeller drives are no longer considered difficult to produce; for this reason the blower cost should not be prohibitive.

Fuel consumption. In this analysis two cases are considered. In the first case the fuel is assumed to be perfectly mixed with the scavenging air by means of some carburetion arrangement, so that no stratification occurs, and any fresh air lost through the exhaust ports carries its fuel with it. This represents about the worst situation possible with respect to fuel consumption. In the second case the fuel is assumed to be injected into the cylinder late enough so that none of it escapes to the exhaust, and only enough is injected to use up the fraction of the scavenging air retained in the cylinder.

In the case of the carbureted gasoline, the fuel used will be equal to the air supplied multiplied by the fuel-air ratio.

$$\text{Lb. of fuel/hr.} = \text{S. R.} \times \rho_s \times V_c \times \frac{\text{r.p.m.} \times 60}{1728} \times F \quad (1)$$

where S.R. = scavenging ratio.
ρ_s = density of fresh air, lb./ft.^{3}
V_c = total cylinder volume, in.^{3}
F = fuel-air ratio.

Horse power output will be equal to

$$\frac{\text{Net b.m.e.p.} \times V_c \times \frac{r-1}{r} \times \frac{1}{12} \times \text{r.p.m.}}{33{,}000} \qquad (2)$$

where r = compression ratio, and other symbols are as before.

Dividing (1) by (2),

$$\text{B.s.f.c.} = \frac{\text{S.R.} \times F \times \rho_s}{\text{b.m.e.p.} \times \frac{r-1}{r}} \times \text{constant}$$

The values of b.s.f.c. to be expected are shown in Fig. 9, assuming a fuel-air ratio of 0.075 which would correspond approximately to a best-power mixture. The rapid reduction in fuel consumption as the engine is throttled on a propeller load curve is due to the lower scavenging ratio required and the consequent reduction in the charge lost through the exhaust ports. At full throttle (constant scavenging ratio) the fuel consumption rises at lower speeds because the b.m.e.p. falls off, while the quantity of fuel per stroke remains constant.

These specific fuel consumption figures are considerably above those obtained with the small four-stroke aircraft engines. Carburetor operation would therefore be unsatisfactory for cross-country flying, except at low outputs, unless some method of stratification were used. For occasional flights of an hour or two near the airport, high specific fuel consumption might not be any more objectionable than it is in an outboard motor boat.

In order to estimate the b.s.f.c. with fuel injection, the indicated specific air consumption was obtained on a C.F.R. engine at the same compression ratio and at several engine speeds. This figure was 6.35 pounds of air per i.h.p. hour. If we assume the same thermal efficiency for the two engines and assume that fuel is added only to the air retained in the cylinder, the net b.s.f.c. for the two-stroke will be

$$\frac{6.35 \times F}{\eta_{mech.}}$$

where $\eta_{mech.} = \dfrac{\text{net b.m.e.p.}}{\text{gross i.m.e.p.}}$.

The calculated b.s.f.c.'s shown in Fig. 9 are in this case quite reasonable, and may be considerably less at leaner fuel-air ratios. From the standpoint of fuel consumption there would thus seem to be no disadvantage in the loop-scavenged two-stroke engine, provided a relatively inexpensive low-pressure fuel-injection system were obtainable. Such equipment is now available for small four-cylinder engines at about one hundred dollars, and would quickly pay for itself in reduced fuel consumption. The first cost, however, appears to be still too high in comparison with the total engine cost.

Compression-ignition operation. Should compression-ignition engines become popular for light planes, the two-stroke engine would be in a still more advantageous position, since all C.I. engines would require fuel-injection equipment. The power output and uniformity of torque would be better than the four-stroke compres-

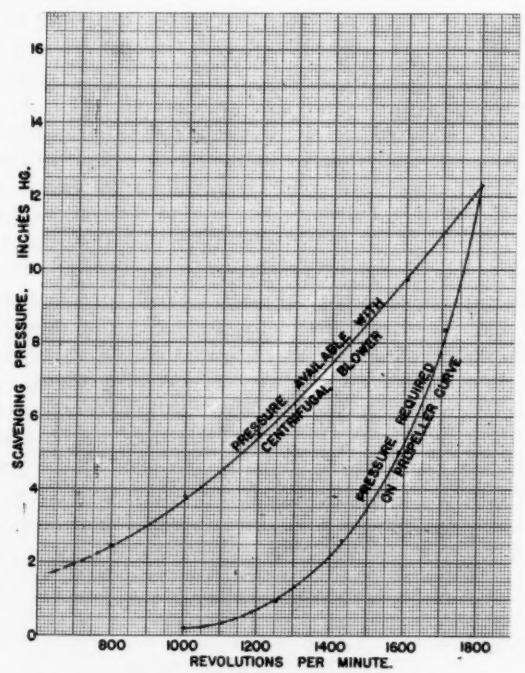


FIG. 8. The effect of engine speed on the scavenging blower pressure required on a propeller load curve, and available at full throttle.

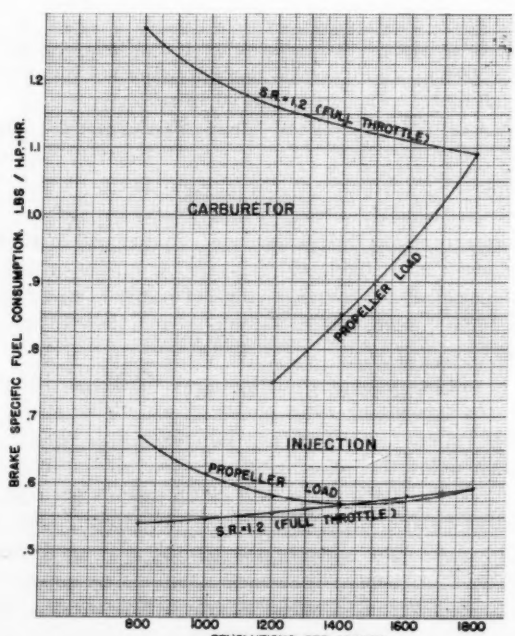


FIG. 9. The effect of engine speed on net brake specific fuel consumption with carburetted and injected fuel.

TABLE 1

Name	Bore and Stroke	V_d	r	B.m.e.p.	B.hp.	R.p.m.	Piston Speed Ft./ Min.	Hp. per In.^{3}	Hp. / $V^{2/3}N^{1/2}$	Lb. per Hp.	B.s.f.c.
Continental 9A-80, 4 cyl	$3^7/_8 \times 3^5/_8$	171	7.5	137	80	2700	1630	0.466	1.63	2.16	..
Franklin 4AC 171, 4 cyl	$3^7/_8 \times 3^5/_8$	171	6.25	118	60	2350	1425	0.350	1.22	3.40	0.58
Lycoming O-145-C, 4 cyl	$3^5/_8 \times 3^1/_2$	144.5	6.50	133	75	3100	1800	0.520	1.71	2.22	0.50
Aircooled Motors, 6 cyl	$4 \times 3^1/_2$	264	7.00	138	120	2600	1520	0.455	1.60	2.38	..
M.I.T. 2-stroke, 1 cyl	$4^1/_2 \times 6$	95.5	7.00	196*	42.5	1800	1800	0.445	2.03	..	0.59
Hypothetical 2-stroke, 4 cyl	3.45×4.6	171	7.00	196*	100	2350	1800	0.585	2.03	..	0.59
Hypothetical 2-stroke, 4 cyl	3.26×4.35	144.5	7.00	196*	89	2480	1800	0.616	2.03	..	0.59

* Calculated on the four-stroke basis for comparison.

sion-ignition engines by about the same ratio as in spark-ignition engines; and the fuel consumption should be just as good, with comparable combustion chambers, as the scavenging is done with fresh air.

Comparison with Four-Stroke Performance

A comparison of the performance of the Massachusetts Institute of Technology engine with present light-plane engines is made in Table 1. This table is based on published data which, of course, may not be strictly up-to-date. The engines are compared on the basis of horsepower per cubic inch only because this is the most commonly used criterion. This practice is theoretically incorrect, as the output of dimensionally similar engines should vary with the square of a characteristic length, l. For non-similar engines the expression

$$\text{Specific output} = \frac{\text{B.hp.}}{V^{2/3} \times N^{1/2}}$$ [6]

where N = number of cylinders.
V = total engine displacement.

is to be preferred, since it assumes that the horsepower per cylinder should vary with an equivalent l^2 equal to (displacement per cylinder)$^{2/3}$. Comparisons are also made in terms of specific output and net b.m.e.p. on the four-stroke basis.

The performance of two four-cylinder two-stroke engines having cylinders similar to the test engine, and running at the same piston speeds, has been calculated from dimensional considerations and is shown in Table 1. One engine has a displacement of 171 cubic inches, the same as the Continental and Franklin; the other has a displacement of 144.5 cubic inches as in the Lycoming. In the case of both two-stroke engines the specific outputs and horsepower per cubic inch are considerably higher than the corresponding four-stroke engines. The relatively low rotative speeds of the two-stroke engines are conducive to good propeller efficiency, without the use of reduction gears.

Conclusions

1. The simple piston-ported loop-scavenged two-stroke engine will have a higher specific output than the corresponding unsupercharged four-stroke engine.
2. Using a carburetor without charge stratification, the specific fuel consumption of this type of engine will be too high for most purposes, at mean effective pressures above 50 pounds per square inch.
3. The specific fuel consumption obtainable with injected fuel will be entirely satisfactory.
4. Because of its mechanical simplicity, this type of engine should be much easier to produce in quantity and should be more reliable in service, resulting in lower first cost and upkeep.
5. Cost and upkeep may be somewhat adversely affected by the necessity for blower and fuel injection equipment, particularly the latter.

References

[1] Rogowski, A. R., and Bouchard, C. L., *Scavenging a Piston-Ported Two-Stroke Cylinder*, N.A.C.A. Technical Note No. 674, 1938.

[2] Rogowski, A. R., Bouchard, C. L., and Taylor, C. F., *The Effect of Piston-Head Shape, Cylinder-Head Shape and Exhaust Restriction on the Performance of a Piston-Ported Two-Stroke Cylinder*, N.A.C.A. Technical Note No. 756, 1940.

[3] Curtis, Charles G., *Improvement in Scavenging and Supercharging Two-Cycle Diesel Engines*, Diesel Power, Vol. II, No. 9, pages 545–549, September, 1933.

[4] *The Scavenging of Two-Cycle Engines*, Sulzer Technical Review No. 4, 1933.

[5] Taylor, C. F., *An Analysis of the Charging Process in the Two-Stroke Engine*, presented at the SAE Meeting, White Sulphur Springs, West Virginia, June 1940.

[6] Taylor, C. F., and Taylor, E. S., *The Internal Combustion Engine*, International Textbook Company, Scranton, Pa., 1938.

On the Technique of Forecasting Low Ceilings and Fog

J. J. GEORGE, *Eastern Air Lines*

SUMMARY

As far as the formation of fog is concerned, air loses its maritime properties after being exposed to sunshine over land for only a few hours. Accordingly, fog which forms over land may be called "radiation" when the trajectory of the air has been mostly continental, and "advection" when it has not been over land during the day preceding the formation of the fog. It is found that radiation fog forms only when the air has been under a cloud cover a part of the previous day, and a forecasting method for such formations is developed. The forecasting of advection fog is shown to be more complex, and various methods necessary in such work are discussed. Examples of each type are presented.

An analysis of the causes of errors being made at present in the forecasting of fog and low ceilings is made, and indicates that substantial improvement in short range forecasts of such conditions is possible at this time. Suggestions for the elimination of present faults are made and a systematic method of forecasting described.

DEFINITIONS

THERE ARE two inadequacies which immediately confront the forecaster who works with fog. First, all definitions of fog refer to horizontal surface visibility and arbitrarily set some value below which fog is said to be present. If the conditions for the condensation of water vapor are present at any place where the wind velocity is low, the condensation will take place near the surface of the ground (except in certain cases where strong inversions already exist above the surface) and fog is said to be present. If the wind is higher, it distributes the cooling and moisture from the surface upward through a thicker layer and the condensation starts at some level above the surface and builds downward. The layer of low clouds thus produced is called stratus. The same processes are present in the formation of both fog and stratus, and the two are forecast in exactly the same way. Especially with reference to aviation forecasting, it is therefore no longer necessary or even practicable to treat fog and stratus clouds as two different problems. Accordingly, the term fog as used in this paper should be construed to include both fog and low stratus clouds.

The second point which must be clarified, has to do with types of fog. "Radiation" and "advection" are the terms which have long been used to classify fog according to cause. The only true advection fogs are, (1) sea fog, (2) arctic steam mists and (3) the fog that occurs when air with a dew point above freezing travels over a snow surface. None of these types occurs over land often enough to be of primary importance.

Most fogs are caused by a combination of factors and this fact has introduced considerable confusion since there is no accepted standard by which to determine the dominant element and classify the fog. For example, many fogs are formed when air passes over a water surface for a time then returns inland where radiational cooling at night causes the formation of a fog. It is apparent that the question, whether the water passage (advection) or the cooling at night (radiation) should be considered the most essential factor, is one that different workers need not interpret the same way.

From an examination in some detail of fog conditions at a considerable number of stations scattered through the eastern United States, a pattern of fog types is discernible which allows a clear delineation between radiation and advection fog to be made.

Pure radiation fog may be thought of as forming with any real density only in air which has had no water history for a minimum of 8 to 16 hours, depending on season, prior to the formation, and forms only when the heating of the surface air layers is restricted during some part of the day by a cloud cover. Minor exceptions occur in deep valleys where air drainage is important and perhaps in mountainous locations by up-slope cooling.

The pure advection types have already been enumerated. To these can be added the entire group of combination radiation-advection fogs since real advection fogs are numerically small over land. Such a fog is easily classified by applying a simple test. If the air in which the fog forms has had even a short path over water since the preceding day, it should be classified in the advection group. It is important to be able to make this distinction since the forecasting problem depends entirely upon the accuracy of the classification.

AREAS AFFECTED

The area over which advection fogs may form is immediately and clearly defined as that portion of the coastal plains which the lower air layers may reach in one night's travel from the water. Snow covered ground regardless of where located also must be included. A practical limit to gradient winds which allow fog to form is about 30 m.p.h., but almost never will this value exist both for the full distance inland and the entire time consumed in traveling this distance; the average transport of air will also be much less be-

Presented at the Meteorology session, Ninth Annual Meeting, I.Ae.S., New York, January 31, 1941.

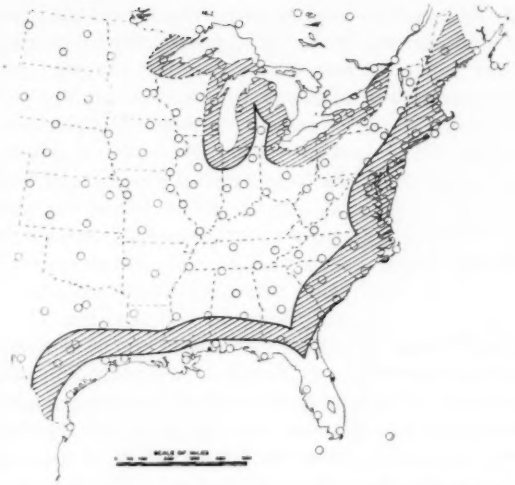


FIG. 1a. Cross hatching indicates the area affected by advection fog during summer.

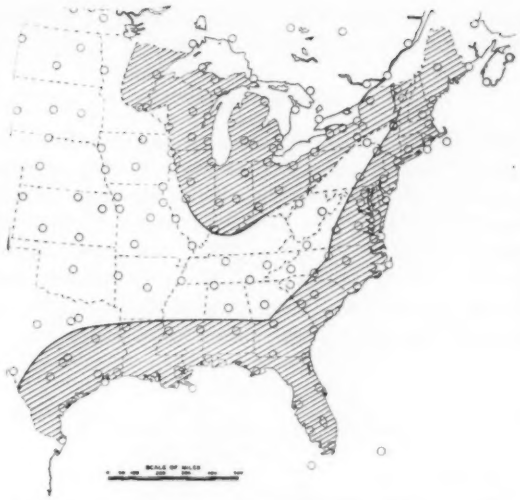


FIG. 1b. Cross hatching shows the area affected by advection fog during fall, winter or spring.

cause of surface friction. In practice, about 200 miles during winter and 150 miles in summer were found to be the inland limits which are exceeded on only the most unusual occasions. This allows a maximum average transport of air amounting to 14 to 18 m.p.h.

In Figs. 1a and 1b the cross hatching indicates the area along the east coast and Great Lakes which is subject to advection fog in summer and during the remainder of the year, respectively. For the most part, these areas are determined by actual studies of different localities, but where the outline is broken, extrapolation is indicated. During summer, there is a strip from 30 to 50 miles wide along the Gulf and Florida coasts in which no advection fog forms. The reason this exists is probably because pressure gradients from sea to land sufficient to overcome the tendency for land breeze formation, allow insufficient time for radiational cooling to take place in this strip. If the land breeze does develop, the accompanying subsidence and other effects are sufficient to prevent the formation of fog. During the colder months, and even in summer farther north, the colder shore waters pre-cool the air, and fog may form even along the coast with reasonable wind velocities. During winter, fall, and spring, the area which is subject to such fog is considerably extended, but it should be noted that this area has nothing to do with the frequency of formation, and for instance, north of latitude 37°, advection fogs are rare in winter.

Radiation fog, of course, may form over almost any portion of the continent. It is, however, undoubtedly more frequent in the area just inland from the advection zones shown in Fig. 2. This is a natural development since a sea to land gradient bearing moist air is ideal to produce nocturnal stratus in the advection zone. Such stratus frequently breaks up into strato-cumulus or cumulus during the day, and clears at night. Both developments restrict surface heating and favor the recurrence of fog farther inland by what can be called the "radiation mechanism."

Along the sea coasts, themselves, within the limits of the land breeze effect, radiation fog is rare. To produce it in this zone requires a solid overcast area for some distance inland. An ordinary day with broken to occasionally overcast clouds may be dismissed entirely as a fog producer along the coast, but if low clouds are solid over a large part of the coastal plain, then clearing in the evening will be followed by fog at

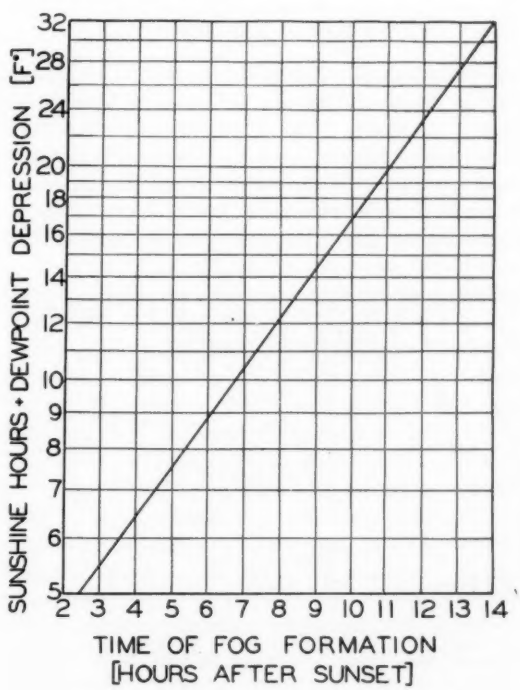


FIG. 2. Forecasting graph for radiation fog.

night even within a few miles of the shore. Such occasions are distinctly unusual.

Classification

The discussion, to this point, has not mentioned the frontal types of fog. The addition of this category completes the classification, and they may now be organized into a table (Table 1) which is essentially that proposed by Willett in 1928. The important difference is in the establishment of rather positive methods by means of which fog may be placed in only one category regardless of who makes the classification.

Table 1

Fog Divided According to Causes

Frontal Types	Air Mass Types
1. Pre-warm frontal	1. Advection (whenever the air in which fog forms has had a recent history over water or snow)
2. Post-cold frontal	2. Radiation (no recent water history)
3. Mixing—radiation	3. Air drainage
	4. Up-slope

A discussion of frontal fog types is beyond the scope of this paper. They comprise a large percentage of the total of forecasting problems, and although considerable progress has been made with them, it is necessary here to confine attention to air mass types only.

Forecasting Radiation Fog

Fogs which form as a result of restricted heating of the lower air layers during the day may be forecast with excellent accuracy. Fig. 3 expresses the forecasting relationship graphically. The difference between the temperature and the dew point at sunset expressed in °F. is added to the number of hours of sunshine occurring during the day obtained from an ordinary sunshine recorder (or estimated if necessary). The time at which the visibility reaches 1 mile in ground fog or at which stratus clouds become overcast is then obtained with reference to sunset. Accuracy of forecasting the time of formation of plus or minus $1^1/_2$ hours may be expected in about 80 per cent of cases. The curve was determined experimentally from Atlanta, Ga., data and has since been checked at various other inland localities. The empirical equation of the curve is easily shown to be.

$$T = 6.13 \text{ Log}_e 0.18 (S + D) + 3.2$$

where T is the time of fog formation in hours after sunset; S is sunshine hours, and D is dew point depression at sunset. The formula of course does not have correct dimensions in this form, but Spilhaus and Arenberg have separately pointed out the possibility of reconcilement of this factor in terms of heat. The formula is predicated upon a constant pressure gradient between sunset and the time of fog formation and since gradient accelerations produce rather large effects on the results, there seems little practical point in further refinement of the formula or of individual measurements until a more accurate method can be devised for caring for wind acceleration.

The conditions for the use of this forecasting curve may be tabulated in this manner:

(1) A material amount of cloudiness other than high clouds must have been present during the day.
(2) The sunset temperature of the dew point should be above 35°F. although the relation is helpful for lower values.
(3) The gradient wind must be near or below 25 m.p.h. at the time of fog formation.
(4) For increasing winds, 1 to 3 hours must be added to the result: for decreasing winds a like amount must be subtracted.
(5) A snow cover may not be present.
(6) The trajectory of the air must show that representative values are being used.
(7) Foehn heating may not be present.
(8) Cloudiness must decrease at night.

Several examples of forecasting radiation fog are illustrated in Fig. 3. A strict application of station data would give correct forecasts for Nashville (NA), Louisville (LV) and Chicago (CG), but not for Indianapolis (ID).

Although Indianapolis has been overcast all day with no sunshine at all, it is apparent from the trajectory that the air which will be present only 2 or 3 hours after sunset will have been exposed to full sunshine during the day (the cross-hatched area represents the overcast). This condition enumerated as numbers 1 and 6 above, indicates no fog will form, and this was the case, visibility never dropping below 1 mile.

At Louisville, the trajectory indicates that the air

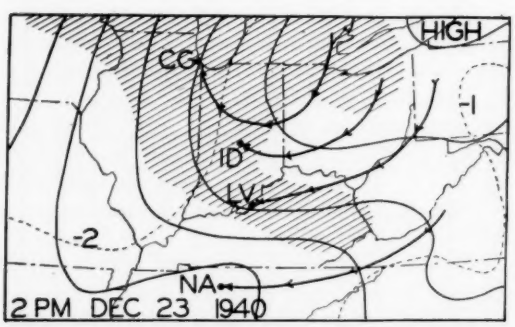


Fig. 3. Section of the weather map for 2:00 p.m., E.S.T., December 23, 1940. Trajectories of lower air shown by stream lines (barbs mark approximate 6 hourly positions of air now at stations indicated). Cross-hatched area is solid overcast; rest is scattered clouds to clear sky. Light broken lines are isallobars.

arriving at the station has been under a cloud cover about 8 hours which means nearly zero sunshine at this time of year. The dew point depression was also very low and the curve would then indicate a time of formation not more than $2^1/_2$ hours after sunset, or near 7:00 p.m. Approximately 8 hours after dark, air which had experienced full sunshine during the day should begin to arrive, and this would indicate a dissipation of fog about midnight. Dense fog formed at 6:30 p.m., became erratic after 10:30 p.m., and finally cleared at 3:30 a.m.

At Chicago, regular conditions exist, but the low clouds did not break until 6:30 p.m. (2 hours after dark). The graph for a 4° dew point depression at sunset and no sunshine would also indicate fog formation within $2^1/_2$ hours of sunset. Modifying factors are the late clearing of the overcast, and the low dew point (32°F.). Both would point to a somewhat later time of formation. The visibility reached 1 mile by 9:00 p.m. and $^1/_5$ mile at 10:30 p.m.

The Nashville trajectory shows little cloudiness present and hence no fog could be expected. None formed.

A more difficult example of the same fog type is illustrated in Fig. 4. The upper portion is a section of the weather map for 1:30 p.m., E.S.T., Jan. 6, 1940, and the lower portion, the same section 6 hours later. In this case, the essence of the forecast is to determine, (1) if the cloud cover will clear with darkness, and (2) if wind velocities will be light enough to permit fog to form. Low clouds prevailed over most of the area at noon with considerable precipitation. The rapid eastward movement of the upper front is frequently sufficient to cause the clearing of skies in this area. Here, in addition, a strong deceleration of gradient is indicated by the isallobars, and since the clouds are supported at least in part by turbulence, this would also aid in the clearing. The dotted line in the upper section indicates approximately the eastward limit of strongly decelerating winds. If this line is extrapolated in the same way that the heavy dashed line in the same section indicates for the movement of the −8 millibar isallobaric center, the area of decreasing winds should then be outlined for 7:30 p.m. The cross-hatched area in the lower map indicates the area in which dense fog formed before midnight; it is interesting to compare the shape of the eastward outline of fog with the shape of the line of wind deceleration.

This forecast is a difficult one. To make it accurately would require meticulous attention to the analysis and drawing of isobars and isallobars. It would then demand a painstaking and time-consuming process of reasoning which, however, could be expected to produce a fairly precise forecast.

The forecast examples given are all taken from the noon weather maps. In practical work, the forecast of such conditions as a rule must be issued by 6 or 7 p.m. in order to be of value. For this reason, the evening map has been used more for the purpose of checking values extrapolated from the noon map rather than as a primary fog forecasting base.

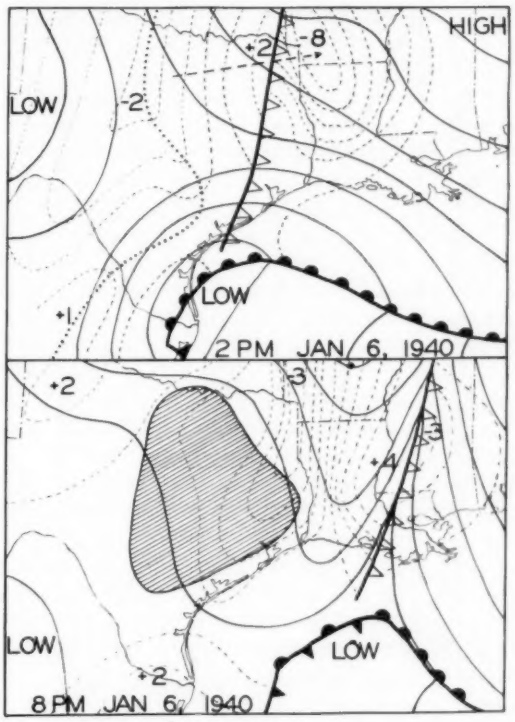


FIG. 4. Top portion is section of the weather map for 2:00 p.m., E.S.T., January 6, 1940; bottom is same section for 6 hours later. Light broken lines are isallobars with amount of increase or decrease indicated at centers. Dotted line in upper portion marks approximate eastward limit of decelerating winds. Heavy dashed line indicates 6 hourly previous movement of isallobaric center. Cross hatching in lower section covers area over which dense fog formed before midnight.

FORECASTING ADVECTION FOG

No universal formula is apparent for forecasting advection fog. Perhaps the outstanding characteristic of this type is that each station must be considered as a separate problem governed by local geography and season.

Figs. 5 and 6 are examples of the forecasting methods which have been found necessary. They are for New Orleans, La., winter and spring, respectively. Notice that the same method is not used in both of them. The relation in all such cases is between moisture which is most conveniently expressed by dew point depression, time of fog formation, gradient wind velocity, and gradient acceleration. The latter quantity has proved rather troublesome but most important. During the winter months, a single curve was sufficient, but with spring, a number of curves indicate that wind direction plays an important part at this station. Stability is also very important, but at present it can only be estimated from the air trajectory at the time it is needed.

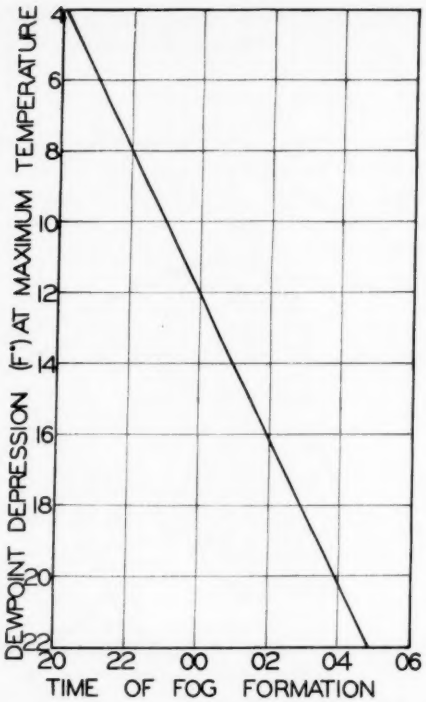


FIG. 5. Example of forecasting graph for advection fog (New Orleans, La., during December, January and February). The curve is for constant gradient winds between 14 and 20 m.p.h. or 9 to 13 m.p.h. increasing at 6:30 p.m., C.S.T. For decreasing winds in this range, fog forms 1 to 4 hours earlier than indicated and for increasing winds, 1 to 4 hours later.

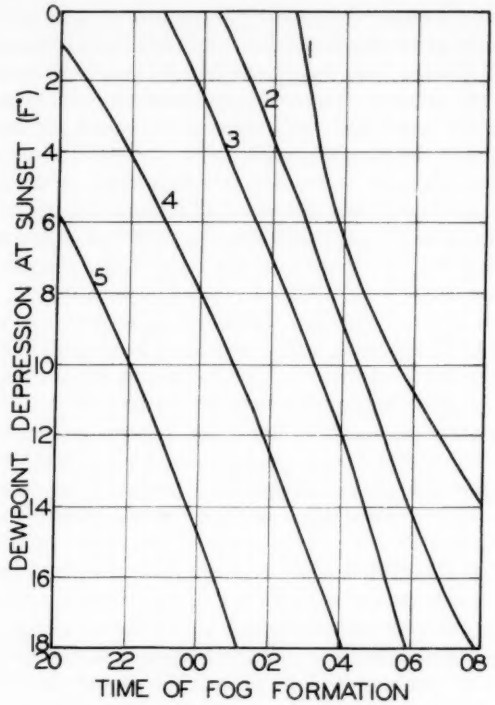


FIG. 6. Another example of a forecasting graph for advection fog (New Orleans, La., during March, April and May). The various curves refer to gradient winds: curve 1—W to SSW, 25 to 30 m.p.h., decreasing; curve 2—W to SSW, 16 to 20 m.p.h., constant; curve 3—W to SSW, 10 to 15 m.p.h., constant; curve 4—S to SE, 13 to 19 m.p.h., constant or decreasing; curve 5—W to SSW, 9 to 12 m.p.h., decreased by 5 to 10 m.p.h., between 3:00 p.m. and 6:30 p.m., C.S.T. All winds given are taken from the 6:30 p.m. weather map by means of a geostrophic wind scale.

Fig. 7 is a section of the 1:30 p.m. weather map for December 10, 1940, and illustrates the method of forecasting advection fog for New Orleans on this night. The dew point depression at maximum temperature was 21°F. which from the curve of constant gradient velocity in Fig. 5 would give a time of fog formation about 4:30 a.m. The isobars on the noon map indicate a geostrophic wind of 19 m.p.h. and the isallobars, a decrease in velocity of about 5 m.p.h. per 6 hours. This would place the wind at 14 m.p.h. at the time of the evening map with a further decrease during the evening. This is about the ideal combination of wind and acceleration to produce early fog and consequently the time indicated by the constant wind curve must be modified to a much earlier hour, say, about 12:30 a.m. In addition, the dew point depression at maximum temperature is fictitiously high because of a rapidly increasing water history for the air during the afternoon and evening which necessitates a still greater correction. The forecast made from this map under service conditions was for fog to form at 10:00 p.m. It actually formed at 8:45 p.m. This is considered an important example because it was the first night of a series with fog, and accuracy is customarily least at such a time, especially when associated with an unusually early hour of formation.

FORECASTING METHODS

When studies such as this had been completed for several stations, and forecasting methods for them similar to the samples given here had been developed, it was felt that impressive gains in forecast accuracy could be expected. What really happened was that mistakes in forecasting were simply spotlighted instead of being excused behind the fiction of erratic behavior which has long been used as a cloak in fog forecasting.

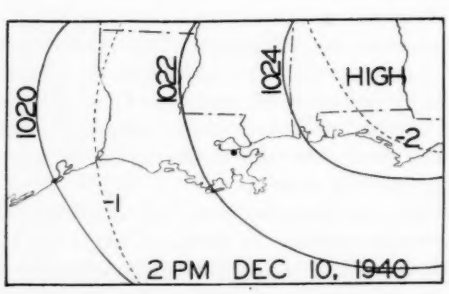


FIG. 7. Portion of weather map for 2:00 p.m., E.S.T., December 10, 1940.

The position was evidently one in which the tools for accurate forecasting were at hand but traditional methods of forecasting were not designed for their use. An analysis of errors in forecasting which were being made with the improved data at hand, led to the conclusion illustrated in Table 2. The values in the table are estimated.

TABLE 2

An Analysis of the Causes of Forecast Errors by Per Cent

Factor	Subdivision	Can Be Eliminated	Cannot Be Eliminated	Total
A. Hurry				
1. Vital factor omitted from consideration	5			
2. Inadequate preparation for forecast				
(a) Careless map work	15			
(b) Poor, or no, computation of pressure and frontal distribution	10			
(c) All pertinent data not collected	5			
		35		35
B. Forecaster mislead by later teletype reports		25		25
C. Wording hedged caused by fear of missing		7½	7½	15
D. Forecast could not logically have been made			10	10
E. Poor judgment in weighting factors			10	10
F. Carelessness		2½	2½	5
Totals		70%	30%	100%

This analysis indicates that about 70 per cent of forecast errors can be eliminated. In addition, experience indicates that such forecasts can be made on a 12 to 18 hour basis which is approximately twice the present airways forecast period. Such a change cannot, of course, be made abruptly, but it can be accomplished over a period of several years.

Some suggestions which might eliminate much or all of the 70 per cent of missed forecasts are:

1. Remove time limits, or allow at least 2 hours more time than at present for forecasting. Under current conditions, most services allow only about one hour for analysis of the data and compilation of the forecast. This amounts to allocating about 3½ hours to stating a very complicated problem and 30 minutes for its solution. In many cases, even when it is properly solved, a hurried wording completely spoils the effect.
2. By pre-analysis, transfer as much work as possible to the period before the map is started rather than after it is finished.
3. Indicate prognostic movements on the finished chart. This is done at present to a large extent.
4. Establish a systematic method of forecasting which might be likened to that used in qualitative chemical analysis. With such a system, the localities concerned can be listed along one side of the form, while across the sheet columns exist which indicate the possibility of various types of weather phenomena occurring at that locality. A careful check of such a form precludes, or at least greatly reduces, the chances of overlooking an important development. Once the possibilities are checked, each column may then be transferred to a work sheet designed to bring out all the factors involved in that particular type of weather at the specific station involved. Such a system has been in use for some time and there is little question but that it improves forecasts and tends to place them on a basis of logical reasoning rather than instinct.

Some advantages possessed by such a system:

(a) It eliminates to a large extent the misleading factor of placing forecasting value on late teletype reports rather than on the weather map.
(b) It reduces hurry errors.
(c) The cause of an erroneous forecast is immediately obvious.
(d) It permits new forecasters to be trained in a fraction of the time required by usual methods.

Flight Level Indicator

ROBERT W. KNIGHT
Civil Aeronautics Administration

SUMMARY

This paper is a report of an experimental instrument designed for the purpose of providing means to effect positive vertical separation between aircraft in flight. From reports of certain air carriers, personnel of the services, and the industry generally, the consensus appears to be in favor of a single, simple, positive method to control cruising altitude levels. To accomplish this, one plan is the employment of non-adjustable, uniform instruments with a common base.

This method of flight altitude control is intended for cruising and holding altitudes only, and suggests a solution to this phase of the air traffic problem.

INTRODUCTION

As TRAFFIC density in the air continues to become an ever-increasing problem because of the rapidly growing numbers of private, commercial and military aircraft, the necessity for positive vertical separation in flight becomes more apparent.

At present the only method of accurately maintaining predetermined cruising altitudes is for pilots to adjust their sensitive altimeters continually as information is received by radio pertaining to barometric pressures at successive communications stations. If, as required for instrument flight, all aircraft were equipped with adjustable altimeters and adequate radio equipment and if all pilots could pay sufficient attention to constant changes, the dangers of collision would be remote. Since this is not the case, however, a more simple and positive plan should prove advantageous.

Thus far very few collisions in the air have occurred, and these have usually been in the vicinity of airports, although numerous accidents have been averted by the quick action of pilots even during conditions of favorable weather. In the past this problem has not been critical because of the limited number of aircraft in use, but it now assumes ever-growing importance with the numerous aircraft now built, building, and on the drafting boards. It was recently estimated by competent authorities that by this year there will be about 100,000 pilots after which the next objective will be 1,000,000 pilots, and that by 1945 the volume of air traffic will be from 20 to 30 times what it is today.

The present system of airway traffic control has been well planned and is capably operated, but growing traffic density on busy airways is becoming a problem of some magnitude. Study is now progressing on a system of automatic posting and recording similar to those in use by brokerage houses. This is a natural and necessary development.

Uniform control of cruising levels and holding levels may serve to simplify operations and to increase safety in controlled zones as well as areas outside such zones.

Civil Air Regulations allow considerable latitude for flight during conditions of contact weather, but hold to rigid standards during instrument conditions. It has been suggested that the same standard "rule of the road" procedure prevail during all conditions of flight insofar as cruising levels are concerned. This would apply to flying on the airways, off the airways or crossing the airways. If this could be accomplished, the present air traffic rules could be greatly simplified, thus assuring closer adherence to regulations by a simple, direct method of instrument observance.

Uniform pressure altitude measurement from an absolute base by a common instrument offers a solution to this problem.

DISCUSSION

The instrument designed to serve this purpose is known as the Flight Level Indicator. The experimental unit illustrated in this paper (Fig. 1) was made for the Civil Aeronautics Administration by the Kolls-

FIG. 1.

Presented at the Radio and Instruments session, Ninth Annual Meeting, I.Ae.S., New York, January 29, 1941.

man Instrument Company. Basically the instrument is an aneroid barometer with typical altimeter mechanism and having a dial graduated like a magnetic compass.

First it is important to emphasize the fact that such an instrument like an altimeter is not actually an indicator of altitude, but is an absolute pressure gage, the scale of which is related to altitude for certain "standard" atmospheric conditions.

Calibration is accomplished by using pressure values determined from a formula* and by assuming that the sea level pressure is 29.921 inches of mercury;† that the temperature at sea level is 15°C. (59°F.), and that the temperature decreases with increase of altitude at the rate of 1.9812°C. (3.566°F.) per thousand feet.‡

Description

The face of the Flight Level Indicator is of the same size as various conventional instruments such as the Altimeter and the Bank and Turn indicator, and is graduated to simulate a magnetic compass.

The dial is marked with the directions, N. E. S and W and is further subdivided in intervals of ten degrees or less.

A pointer attached to the aneroid mechanism indicates the direction in which the flight should be proceeding at that altitude. The pointer makes a complete revolution (360°) for every change of 2000 feet in nominal altitude. Clockwise movements of 90°, 180° or 270° indicate increases of 500 feet, 1000 feet and 1500 feet, respectively. Counterclockwise movements indicate corresponding decreases in altitude.

There is a direction marker on the outer edge of the dial which can be set manually to the desired compass reading. This is merely for convenient reference to the compass bearing being flown.

The instrument is provided with a static connection for atmospheric pressure and an electrical connection for illumination. Rim lighting may be provided in the conventional manner with a 3 volt electric lamp. It weighs 1.17 pounds and is designed for mounting on the instrument panel.

A subdial, which is visible through an aperture below the center of the main dial, indicates flight levels by numbers. Each cardinal heading or 500 foot level of nominal altitude is designated as a flight level and given a number. For example, sea level, "W," under standard atmospheric conditions is "flight level" O; 500 feet, "N," is flight level No. 1; 1000 feet, "E," is flight level No. 2; 1500 feet, "S," is flight level No. 3; 2000 feet, "W," flight level No. 4, etc. (actual altitude depending upon existent atmospheric pressure) above sea level.

As altitude is increased or decreased the subdial rotates slowly and indicates, against a reference index on the fixed dial, the number of the level, or the nearest level, on which the airplane is being flown. There are 32 flight levels totalling 16,000 feet of nominal altitude, in the range of the instrument.

Thus flight levels are automatically related to the appropriate magnetic compass bearings in accordance with the formula:

$$L = \frac{C + 90}{90} + 4n$$

where

L is the number of the flight level
C is the magnetic compass bearing in degrees and
n is the number of revolutions made by the pointer.

For example:

$$C = 0° \text{ (north)}$$

Therefore during the first 2000 feet or first revolution of the pointer from sea level

$$L = \frac{0 + 90}{90} = 1$$

during the second 2000 feet

$$L = \frac{0 + 90}{90} + (4 \times 1) = 5$$

and during the third 2000 feet

$$L = \frac{0 + 90}{90} + (4 \times 2) = 9$$

etc., for northerly headings.

Therefore, levels 1, 5, 9, etc., are automatically used when flying on northerly bearings; 3, 7, 11, etc., on southerly bearings; 2, 6, 10, etc., on easterly bearings; and 4, 8, 12, etc., on westerly bearings. On intermediate headings, intermediate levels are to be used. For example, when flying a magnetic bearing of 60°, and with the rotating pointer of the indicator on that course, a level $^2/_3$ of the way between levels 1 and 2, or between 5 and 6, etc., would be automatically used.

Inasmuch as the instrument is of the same basic design as conventional altimeters, and even more simplified types may be developed later, no detailed description of the mechanism is included in this paper.

---

* This formula is

$$H\ 221.152\ T_{ma} \log \frac{P_0}{P} H_0$$

where

H is the altitude in feet above sea level
T_{ma} is the mean temperature, in degrees Centigrade absolute, of the air from the ground to the altitude, H
P is the pressure at altitude H
P_0 is the pressure at the ground
H_0 is the altitude of the ground over sea level

For altitude in meters the formula is

$$H\ 67.4073\ T_{ma} \log \frac{P_0}{P} H_0$$

† 760 mm. of mercury or 1015 millibars.

‡ 6.5°C. per thousand meters.

Naturally, to be effective all such instruments in all airplanes must indicate from a common zero, consequently, as mentioned elsewhere, there is no method of making barometric adjustments in flight. Common zero is obtained by setting the pointers of all instruments at west (W) for a pressure of 29.921″ Hg. All instruments must, of course, be calibrated accurately, and periodic inspections will be necessary. Thus, the flight levels of different airplanes will be exactly related even though the barometric pressure changes. Although the entire series of flight levels may vary with respect to altitude measured from the earth, they remain uniformly spaced in proper relation to one another.

Application

Before discussing the general use of the Flight Level Indicator it is important first to emphasize the following points:

1. It is not adjustable.
2. No correction for air temperature is required.
3. While its purpose is the vertical separation of aircraft, it can also be used as a check for the altimeter.
4. It does not replace the altimeter which should be used, as always, for the determination of altitude above the ground.
5. It would be necessary that all aircraft flying over continental United States be equipped with these instruments just as automobiles must be equipped with such well known safety devices as windshield wipers. In the event such a system was adopted, it might be permissible for all aircraft to fly at any desired level or direction while below an altitude of 2000 feet above terrain during class "C" (contact) weather.

As at present, westerly headings would be flown at even altitudes and easterly headings would be odd altitudes. However, a northerly heading would be a level 500 feet above the nearest one for a westerly heading and 500 feet below the nearest one for an easterly heading. A northwesterly heading would be flown midway between the nearest level to the west and the nearest one to the north or 250 feet above a west heading and 250 feet below a north heading.

Theoretically 36 aircraft with headings 10° apart could cross over a given point simultaneously within an altitude range of 2000 feet without accident. There would still be the remote possibility of collision caused by an overtaking aircraft flying exactly the same track and level as the one ahead, but forward visibility is usually sufficient to preclude such an occurrence, and airway traffic-control is alert to this problem.

Aircraft flying in opposite directions regardless of course would have vertical separations of 1000, 3000, 5000 or 7000 feet, etc. Aircraft crossing an airway at 90° would be separated vertically from those flying the airway by 500, 1500, 2500, or 3500 feet, etc.

Along the airways, the particular flight level to be flown (consistent with heading) would be assigned by airway traffic-control centers, and included in flight plans. Holding altitudes flown over fixed points while awaiting permission to descend and land during instrument weather, would be prescribed in terms of flight level numbers and closer spacing than at present might be possible with safety, thus reducing landing delays.

The use of the instruments would be applicable only for air space at reasonable distances from airports, except at higher altitudes, because of climbing and descending aircraft in those vicinities.

To use the instrument it is necessary only that each pilot climb to his desired or assigned cruising altitude and then maintain the pointer in agreement with the magnetic bearing being flown. The compass is the reference. For example: If a pilot desired to fly at an altitude of approximately 6000 feet on the airways from New York to Harrisburg *via* Philadelphia by contact or instruments his procedure would be as follows:

1. Before taking off, or soon thereafter, he would set the index marker (for ready reference) to 233° (assuming he had not calculated for drift), which is the magnetic bearing from New York to Philadelphia.
2. After taking off he would climb until the sub-dial reading indicated that he had passed level 11 (nominally 5500 feet), and continue climbing until the rotating pointer reached the index marker, or 233°. At this point he would level off and maintain this reading. If he found it necessary to fly a bearing of 245° to maintain the desired track because of drift he would climb slightly until the pointer reached 245° adjusting the index marker to this bearing if he desired.
3. Upon approaching the Philadelphia range he would alter his course to 293°, which is the magnetic course from Philadelphia to Harrisburg, and climb until the pointer agreed with the magnetic bearing being flown and maintain that reading.

Throughout the flight he would merely maintain the pointer of the flight Level Indicator in agreement with the magnetic bearing being flown.

If, for some reason, he desired to fly at a higher or lower altitude after being under way, he would climb or decend until the pointer of the Flight Level Indicator made one or more complete revolutions (2000 feet, or 4 levels, or multiples thereof) and again agreed with the magnetic bearing. The pointer turns clockwise as the airplane gains altitude and counterclockwise as it loses altitude, just as does the altimeter itself.

It is as easy to maintain the pointer of the Flight Level Indicator on the magnetic bearing being flown as it is to maintain a reading of the altimeter.

The Human Element

There is, of course, the ever-present possibility that certain pilots would disregard the instrument and fly at altitudes of their own choosing. It is believed, however, that the simplicity of the proposed procedure would assure its being followed. In the interest of

safety pilots would use it, just as one drives an automobile on the right side of the highway.

Check for Altimeter

The Flight Level Indicator may be used as a check of the regular sensitive altimeter carried in the airplane. The procedure is as follows:

1. Set the barometric scale on the altimeter at 29.92 and read the indicated altitude.

2. Multiply the flight level observed from the subdial of the flight Level Indicator by 500 to obtain the nominal altitude in feet. The reading of the altimeter should correspond to this computed altitude.

Such a check would be particularly valuable on an airplane carrying two altimeters. When two instruments differ either may be wrong.

Adaptation to Automatic Pilot

The Flight Level Indicator can be adapted to the gyro autopilot as a means of automatically controlling the altitude being flown. By installing the instrument as a part of the autopilot system the manually controlled course indicator marker would be interconnected to the autopilot in such a way that when the marker is set at the desired course (and hence the desired altitude or flight level) the autopilot will bring the airplane to this level and keep it there.

Conclusions

1. Flight Level Indicators, uniformly measuring pressure altitudes above a nonadjustable base, provide a solution to the universal problem of maintaining cruising and holding altitudes.

2. Relating flight levels to compass directions assures maximum vertical distance between airplanes on intersecting courses.

3. Relating flight levels to compass directions provides a simple easy rule for pilots to follow.

4. The use of this instrument would permit great simplification of air traffic rules. Fewer and simpler rules mean less violation and greater safety.

5. The absence of any adjustments eliminates the possibility of incorrect settings.

6. Even though all pilots do not maintain the pointer in exact agreement with the magnetic compass, the danger of collisions will be substantially reduced.

7. While the Flight Level Indicator may be used to check the altimeter, it does not take the place of the altimeter. The function of the altimeter is the maintenance of terrain clearance while that of the Flight Level Indicator is the maintenance of interplane clearance.

8. The cost of such instruments built in volume would be relatively low.

9. The weight of 1.17 pounds and space occupied in the instrument panel are justified.

10. The Flight Level Indicator may be used in connection with the autopilot as a means of automatically controlling cruising altitudes.

Unit Method of Beam Analysis*

F. R. SHANLEY AND F. P. COZZONE
Lockheed Aircraft Corporation

SUMMARY

Various improvements in the methods of analysis for determining axial and shear stresses in box beams have been combined into a practical tabular method which permits a considerable saving of time. The method also eliminates some of the errors often made through injudicious use of the classical methods of analysis and offers a means of extending the process to include secondary effects, such as caused by "shear lag" and cutouts.

INTRODUCTION

IN AIRPLANE structural analysis the "beam theory" is commonly employed in the analysis of *hollow* (box) beams, as contrasted with the *solid* or concentrated type of beam for which the theory was originally developed. In dealing with a solid beam, such as a rectangular wooden spar, the usual method of computing the so-called "longitudinal" shear stress

$$f_s = \frac{SQ}{bI} \tag{1}$$

is convenient and rational when applied to a rectangular beam of this type (reference 1). However, when dealing with a box beam such as used in airplane wings we find conditions which indicate a different method of analysis that is more accurate in the general case. The two types of beams are indicated in Fig. 1.

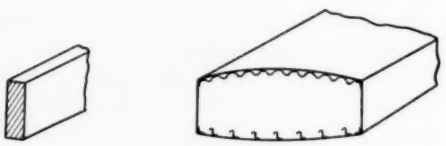


FIG. 1.

One of the fundamental errors involved in applying the classical formula for shear stress to the box beam occurs when the beam has taper in width. Eq. (1) was derived for a beam of constant cross-section and does not give a true picture for the beam with a variable cross-section. The nature of this error will be further discussed in the development of the unit method. Another feature of the more common methods of analysis is the necessity for computing the principal axes of the beam before the axial stresses can be determined. This becomes particularly laborious in the initial design stages when successive attempts are being made to arrive at the most efficient distribution of flange material. ("Flange" material is here intended to represent the material which transmits axial stresses due to bending.) It is obviously desirable to eliminate the computation of the principal axes and this step has therefore been included in the method. Another feature that is automatically covered is the effect of taper in depth of the beam. In the usual method of analysis it is necessary to make a separate allowance for the shear load resisted by the vertical components of the axial loads in the flanges. An accurate analysis by this process would become very complicated for certain types of beams in which the slopes of various flange members differed appreciably.

BASIC PRINCIPLES

Since there is nothing essentially new in the various methods of attack that have been utilized in working up the tabular procedure, attention will be centered first on the method of accounting for the effects of taper in beam width. The importance of this problem does not seem to have been generally recognized, particularly in connection with the attempts that have been made to evaluate the effects of "shear lag." (Such work is too often based on a beam of constant width and depth.) To illustrate the nature of the error involved in using the classical formula for shear distribution, when applied to a beam tapered in width, we can take the idealized case of a constant depth beam with a triangular plan-form carrying a constant shear load, as indicated in Fig. 2. It is apparent that the axial load, P, in the flanges is equal to the bending moment divided by the depth. Since the bending moment is equal to the product of the shear load and the distance through which it is transmitted, we find that the axial load P is directly proportional to the distance from the small end of the beam. If we assume that the flanges of the beam have a constant thickness, it follows that the flange area is likewise proportional to the distance from the end of the beam. The top flange of the beam can therefore be visualized as in Fig. 3, in which the increments of flange load are indicated by the loads ΔP introduced by the shear webs. By dividing the flange material into lengthwise strips of equal width it becomes obvious that the axial load in each strip is the same. It therefore follows that there can be no shear load transmission across the flange material, as this would require a difference in the axial load carried by adjacent strips.

Received November 27, 1940.

* Published with the consent of the copyright owners, Lockheed Aircraft Corporation.

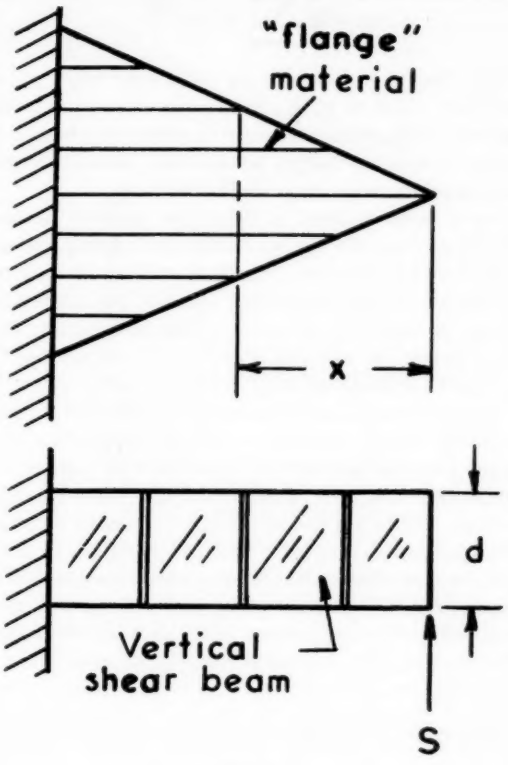


FIG. 2.

The distribution of shear load in the beam of Fig. 3 can therefore be illustrated as in Fig. 4a in which the shear flow q is plotted for a cross-section of the beam. Fig. 4b shows the shear flow for the same cross-section as computed by the classical formula given in Eq. (1). For this idealized case it is apparent that the classical formula gives entirely misleading results by indicating a shear flow across the beam width that cannot possibly exist under the assumed conditions.

In an airplane wing it is quite possible for the conditions to approach more closely those of the triangular beam than those of the rectangular beam. Most of the work done in estimating the effects of shear lag must therefore be regarded with caution when applied to a practical case, as the shear flow conditions are obviously greatly affected by the taper of the wing beam in width.

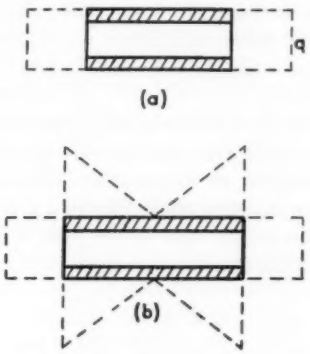


FIG. 4.

In fact, the optimum design from the standpoint of efficiency would be a wing beam in which the width and depth were varied in such a manner as to produce a constant "unit" flange load, that is, a constant axial loading per unit of flange width. Such a beam would have no "transverse" shear flow (due to "vertical" shear loads) and therefore no shear lag.

To eliminate the error involved in applying the classical formula Eq. (1) it is possible to utilize an elementary principle that is probably already evident from the discussion of the triangular beam and which is the consideration underlying the deduction of the classical formula. This principle is simply that the shear flow between any two adjacent spanwise strips (units) of flange material must depend on the *variation* in axial load within the strips. For example, consider a unit such as indicated in Fig. 5, which represents a top view of the compression flange of a wing beam. The unit of flange material is determined by a unit width b and a unit length x. We can indicate the axial loads

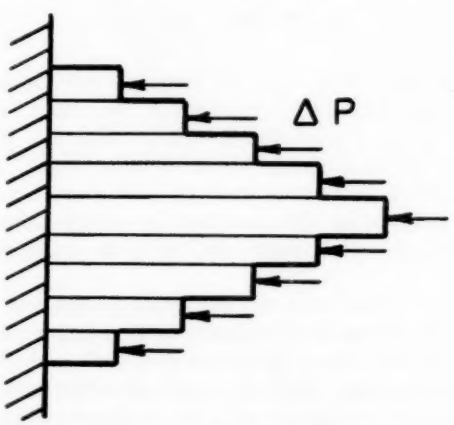


FIG. 3.

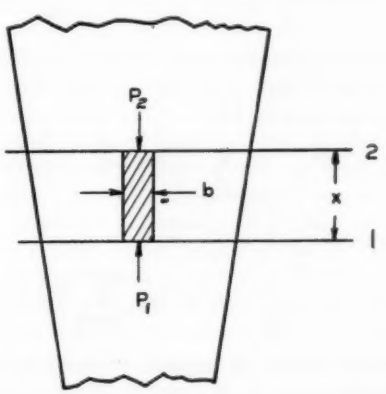


FIG. 5.

acting on this unit by the terms P_1 and P_2. It is now obvious that in order to maintain equilibrium there must be shear forces acting on the edges of the unit. If, for instance, we assume that the shear force on one edge is zero, the shear force on the other edge must of necessity become equal to the difference in the axial loads acting on the ends of the unit. This can be expressed by the simple equation

$$S_{1,2} = P_2 - P_1 = \Delta P \tag{2}$$

This equation gives the total shear load required to put the unit in equilibrium and in considering the reasonably short lengths we can assume that this load is distributed over the edge of the unit in a uniform manner. From this assumption the "shear flow" q is determined from the equation

$$q = \frac{S}{\Delta x} = \frac{\Delta P}{\Delta x} \tag{3}$$

Shear flow is obviously defined as the shear load per unit of length and could therefore be called the "running" shear load. To obtain the corresponding shear stress it is of course necessary to divide the shear flow by the thickness of the material involved.

The problem of shear flow analysis can now be attacked by this elementary principle and it becomes possible to eliminate entirely the use of the classical formula, Eq. (1), thereby eliminating the usual conception of static moment, Q, and the use of the shear load, S. This can be better understood by realizing that once the axial (or "normal") load distribution in any structure is completely determined, the shear flow distribution must correspond. This relationship may or may not be sufficient to determine the shear flow completely, depending on the degree of redundancy of the structure in question.

It can be seen that the classical formula of Eq. (1) is simply a special application of the present method. This can be shown by writing the equation for difference in axial load, Eq. (2), as

$$\Delta P = P_2 - P_1 = \frac{M_2 y_2}{I_2} A_2 - \frac{M_1 y_1}{I_1} A_1$$

Then assuming that the shear flow at the "outer fiber" of the beam is zero, the flow at any "longitudinal" cross-section is given by

$$q = \frac{\Delta P}{\Delta x} = \frac{1}{\Delta x} \sum \left[\frac{M_2 y_2 A_2}{I_2} - \frac{M_1 y_1 A_1}{I_1} \right]$$

When $y_1 = y_2$, $A_1 = A_2$ and $I_1 = I_2$

$$q = \frac{1}{\Delta x} (M_2 - M_1) \sum \frac{yA}{I}$$

Since the change in bending moment $(M_2 - M_1)$, over the distance Δx, is equal to $S(\Delta x)$ this equation reduces to

$$q = \frac{S}{I} \sum yA = \frac{SQ}{I}$$

Note that this derivation is valid only when y_1, A_1 and I_1 are equal to y_2, A_2 and I_2, respectively. In the case of a beam which is tapered in width the above conditions of equality cannot be satisfied; hence the error introduced by using the classical formula.

For a beam tapered in *depth* the classical formula fails to account for the fact that the bending stresses at each end of the unit length, x, will be affected not only by the change in bending moment but also by the change in depth of the beam. This also produces an error, unless the value of S is modified to account for the "vertical components of the flange loads."

Summing up, it is evident that the classical formula for shear flow is actually a special application of a simple principle and that its derivation is based on the assumption that the beam does not vary in cross-section. When applied to a wing or fuselage structure this assumption is usually violated, resulting in errors that may be of considerable magnitude and which are especially misleading in dealing with "shear lag" problems. The obvious answer is to go back to the elementary principle and use it directly.

Shear Flow Distribution (Single-Web Beams)

From the foregoing principles it is evident that the relationship between the shear flow of adjacent flange units may be determined from the axial loads in the flanges. For an "open" beam, such as an I-beam, channel or any "single-web" type (Fig. 6), the shear flow at the "free" edge must be zero. For a given distribution of axial stresses the shear flow may therefore be completely determined by starting from one edge

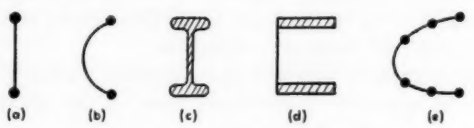


Fig. 6. "Open" type of beams.

with a shear flow equal to zero and calculating the shear flow between adjacent units. For such "single-web" beams a given distribution of shear flow will be equivalent to a certain resultant force, which can be determined by various methods (references 2 and 3). The location of this resultant is usually called the "shear center," "elastic axis" or some other similar name.

If we assume that the individual flanges and shear webs are incapable of resisting torsion (approximately true in most cases) it follows that such a single-web beam can resist a shear load *only* when the load is applied so as to coincide with the resultant of the shear flow, *i.e.*, the "shear center." For example, the beam

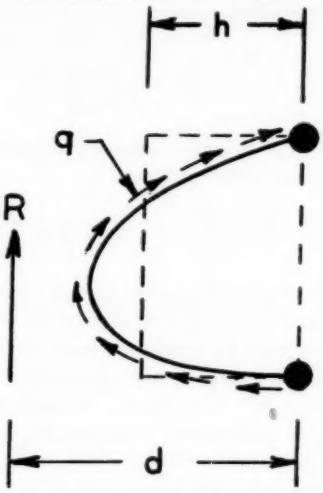


FIG. 7. Resultant of shear flow.

of Fig. 7 is unable to resist a shear load unless the load is applied at the shear center, located at a distance d from the line joining the two flanges. (In this case d is equal to twice the average height h of the enclosed area where h is measured from the chord line joining the flanges as above.) It should always be kept in mind, however, that the accuracy of the shear center location will depend entirely on the accuracy with which we can calculate or estimate the axial stress distribution due to bending. Local effects, end conditions and "shear lag" may modify the distribution obtained from the classical beam theory, thereby causing a corresponding modification of the shear flow and location of the true shear center.

SINGLE-CELL BEAMS

The "single-web" beam cannot usually be used by itself as a practical structural member, due to its inability to resist torsion. It is therefore customary to use such beams in pairs (as in a two-spar wing) or to form closed sections by adding one or more webs. If only one web is added, the beam may be called a "single-cell" beam (Fig. 8). The shear flow distribution in such a beam is not determined by the bending stresses alone, as it is impossible to find a point of zero shear flow that will be independent of the external loads.

FIG. 8. Examples of single-cell beams.

Several different methods of attack may be used to determine the shear flow, but all are based on the principle that the external loads and internal stresses must be in equilibrium. For our purposes it is most convenient to use this principle directly, which can be done by assuming an unknown value for the shear flow q at any convenient starting point, instead of starting with zero as in the case of the single-cell beam. The shear flow at all other points around the cross-section of the beam may then be determined in terms of this unknown value of q by working with the unit axial load as previously described. Finally we can find the moment (torque) of this shear flow about any convenient reference point and equate it to the moment of the external loads about the same point, thus determining the value for q at the starting point. Values of q at all other points are then determined from previously calculated relationships.

Exactly the same results will be obtained if we hypothetically "cut" the cell web at any point and thereby assume the shear flow at that point to be zero. The shear flow around the entire cell may now be determined as for a single-web beam. If, by chance, the resultant of this shear flow coincides with the resultant of the external shear forces our answer will be correct. If it does not, there will be an unbalanced couple, represented by the external and internal shear loads, that must be put in equilibrium. This is done by treating the cell as a torsion box and applying the usual formula

$$q = \frac{T}{2A} \tag{4}$$

where T is the unbalanced couple or torque and A is the area of the entire cross-section enclosed by the shear web (reference 3).

"ELASTIC AXIS" AND "END FIXITY"

This method of obtaining the shear flow eliminates the use of an "elastic axis," thereby removing the difficulty of defining and locating this elusive thing. As pointed out by Hatcher (reference 4) and by the senior author (reference 3) the shear flow in a single-cell beam is not a statically indeterminate problem, if it is assumed that the axial stresses due to bending have been previously determined. It must of course be recognized that the present method does not account for the effects of "end fixity," such as are found at root sections of an airplane wing. Such fixity violates one of the implied assumptions on which the solution is based; namely, that cross-sections are free to warp under torsional loads. The "elastic axis" method also fails to solve this problem, which is best attacked by supplementary analyses. However, the effect is confined largely to a local region near the root of the wing. From a practical standpoint it will usually be found that end fixity reduces the shear flow in the portions of the wing structure critical for shear loads. This is accompanied by an increase in some of the axial stresses due to bending. Fortunately, critical wing design conditions are such that maximum torsional loads seldom

occur simultaneously with maximum bending loads. The present methods are therefore likely to be found conservative when the entire picture is considered.

Chordwise Shear and Bending

Since the total axial stresses in the flange material represent the combined effects of bending in the vertical and horizontal (chordwise) planes, it follows that by using "total" bending stresses the unit method of shear flow distribution accounts for the shear loads in both planes. The shear forces due to chordwise loads are thereby taken care of with no additional labor, in contrast with methods which attempt to handle them separately.

Effects of Taper in Beam Depth

In the case of a simple beam such as shown in Fig. 6a the effects of taper in depth are easily determined. Fig. 9 indicates one simple method by which the shear load resisted by axial loads in the flanges may be computed. Another method consists in calculating the

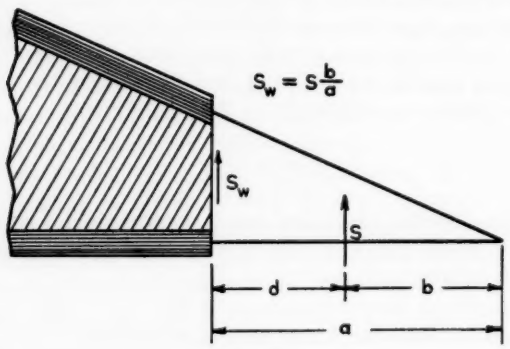


Fig. 9.

flange loads required to resist the bending moment and then determining the "vertical" components of these flange loads. As previously explained, the "unit" method of analysis also takes care of this effect. This can easily be proved mathematically by considering a "unit" of a tapered beam and developing the general equations for the shear flow, as determined by both methods. Using the case illustrated by Fig. 10, we find that the shear flow at the mid-section of the unit (at h) is given by the two equations:

$$\text{(old method) } q = \frac{1}{h^2}\left[Sh - Sd\,(\tan\theta + \tan\alpha)\right] \quad (5a)$$

$$\text{(unit method) } q = \frac{1}{h_1 h_2}\left[Sh - Sd\,(\tan\theta + \tan\alpha)\right] \quad (5b)$$

(See appendix for derivation)

The ratio between the shear flow determined by the new and old methods may therefore be expressed by the factor

$$K = \frac{q_{new}}{q_{old}} = \frac{h^2}{h_1 h_2}$$

which can be written

$$K = \frac{1}{1 - \frac{1}{4}\left(\frac{\Delta h}{h}\right)^2} \quad (6)$$

where $\Delta h = h_2 - h_1$ is the change in height of beam per unit length used in the analysis

$$h_1 = h - \frac{\Delta h}{2}, \qquad h_2 = h + \frac{\Delta h}{2}$$

and h = the mean height of the unit in question.

It is evident that K will always be greater than unity for any case in which the beam tapers so as to have increasing depth in the direction of increasing bending moment. The "unit" method is therefore conservative for such cases, which are the most common ones. Eq. (6) also indicates that the correction is negligibly small for the usual case of a tapered wing beam. For example, if a unit length of 30 inches is used for a beam which is 20 inches deep at the center of the unit in question, and if the change in height in this unit length is 1 inch, the correction involved is less than one-tenth of one per cent. The equation for K also indicates that it is consistent to use increasingly larger unit lengths as the size of the beam increases, a feature that tends to keep the computations from becoming too voluminous as the size of the wing is increased.

External Loads

To build up a tabular method of computation that can be handled by inexperienced personnel it is necessary to have a clear definition of symbols, conventions, directions, etc. It is also essential that the external loads be specified in a standard manner. It will be assumed, in this case, that all "external" loads for the cross-section in question are converted into resultant loads and torques acting at a point on a horizontal reference axis, previously determined. (This is standard practice at Lockheed and is more fully described in reference 5.) The reference axes used are illustrated in Fig. 11. The resultant loads and torques are graphically indicated as shown opposite Fig. 11b. This figure also defines "positive" values and directions. (Note that left-hand rule is applied to show moment vectors.)

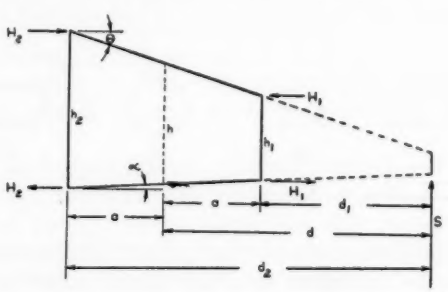


Fig. 10.

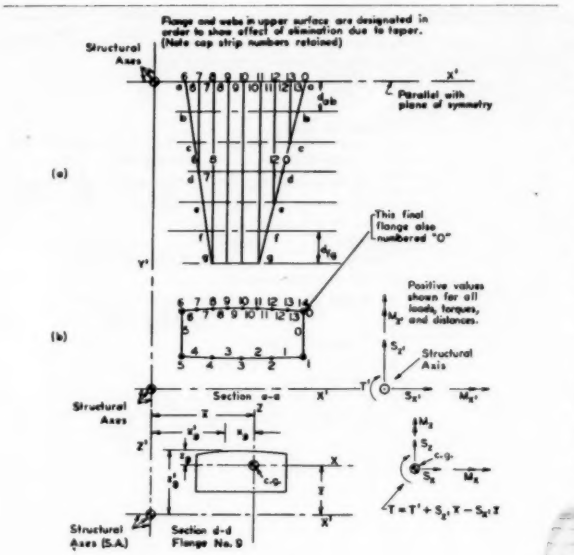

Flange and webs in upper surface are designated in order to show effect of elimination due to taper. (Note cap strip numbers retained)
Structural Axes
Parallel with plane of symmetry
This final flange also numbered "0"
Positive values shown for all loads, torques, and distances.
Structural Axis
Section a-a
Structural Axes
Structural Axes (S.A.)
Section d-d Flange No. 9
c.g.
(a)
(b)

FIG. 11.

Before the bending stresses can be computed it is necessary to determine the centroid of the cross-sectional "flange" material and the moments of inertia of this material about two reference axes through the centroid.

As shown in Fig. 11, cross-sections are taken normal to the horizontal (y) reference axis, *i.e.*, parallel with the plane of symmetry. These sections are taken at any suitable point, preferably at constant intervals (d). In plan view the wing is treated as though no dihedral were present, that is, the projected view is used. This method is employed in order to simplify the procedure. When the dihedral angle is relatively large, corrections can be made as indicated in reference 3. It is of course possible to rotate the reference axes to eliminate this error, if desired.

The beam structure is now further subdivided by "longitudinal" planes as shown in Fig. 11a. To facilitate computations, flange material is assumed to be concentrated into effective "units," which coincide with the spanwise divisions. (When buckling of the skin is to be accounted for the units must include the "effective width" of flat sheet.) Units are numbered consecutively around the section. The "web" material between flange units is similarly numbered. Note that the flange and the web immediately adjacent to the flange in a clockwise direction are given the same number. Numbers are assigned to the *largest* cross-section of the beam and are maintained for a given spanwise unit, dropping out numbers of those flanges and webs which disappear as the beam narrows toward the tip.

DETERMINATION OF CENTER OF GRAVITY AND MOMENTS OF INERTIA

In Tabular Form 1 are entered all the values of cross-sectional area for each flange, together with the horizontal and vertical distances from the reference axes. (Columns 1 to 4.) *These are the only values that must be obtained from the drawing* (with the possible exception of some correction factors that might be desired in certain cases, to be explained later).

In the upper right portion of the form appear calculations for $\bar{x}$ and $\bar{z}$ (the horizontal and vertical distances, respectively, of the c.g. from the reference axes). These values are obtained as the result of the division of the sum of first moments of area about the reference axes divided by the sum of the flange areas.

Having determined the location of the center of gravity, the distances of the flange units from new reference axes through the c.g. are found in columns 5 and 6 (see Fig. 11c). These new axes are parallel to the original reference axes.

At the bottom of columns 5 and 6 appears a reference to an arithmetical check shown in the upper right portion of the form.

Columns 7 and 8 cover the computation of first moments of area about the axes through the c.g. and a check reference is made at the bottom of these columns.

The computations of moments of inertia and product moment of inertia (hereafter called second moments of area and product moment of area, respectively) are covered in the upper right portion of the form.

Column 13 is used as a check on the values of I_x, I_z and I_{xz}.

Columns 9 to 12 are used to determine another property of the cross-section which will be used later in determining shear flow.

MOMENT OF SHEAR FLOW

To explain the tabular method of computing the moment due to shear flow consider the web unit shown in Fig. 12. We can assume that the actual web is curved as shown by the solid line 2-3 and that the shear flow in this web unit (No. 2) is q_2. The moment of this shear flow about the c.g. may be found by multiplying the shear flow by twice the area enclosed by lines 0, 2, 3, 0, indicated by the shaded area in Fig. 12. The value of twice the area determined by the straight lines from 0 to 2, from 3 to 0 and the *straight* chord line from 2 to 3 is determined by differencing the two simple operations indicated in columns 9 and 10, respectively,

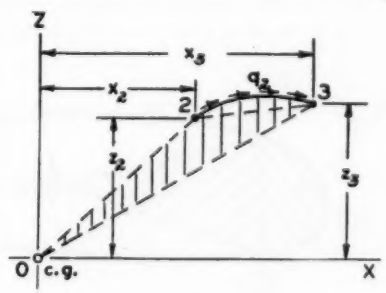


FIG. 12.

TABULAR FORM 1 SECTION PROPERTIES AT STATION _____ , _____ IN UPPER SURFACE

MODEL _____ PG. _____ REF. _____

REF. CODE (col) (row)

No. of Dec. Places for Col. Appears	Op. Code Wkn.	Last Rev. Date Wkn.	Released for Printing	Check	Rev.	Dates	Single Cell: Number Flanges clockwise beginning with lower right. Two Cell: Number Flanges clockwise beginning with Flange at lower end of common web.	1 Flange or Web n	2 Area Of Flange A	3 Hor. Dist. from Z' Axis	4 Vert. Dist. from X' Axis	5 DISTANCE FROM C.G. Hor. x̄	6 DISTANCE FROM C.G. Vert. z̄	7 FIRST MOMENTS ABOUT C.G. Hor. Az̄	8 FIRST MOMENTS ABOUT C.G. Vert. Ax̄	9 CROSS PRODUCTS $x_{n+1}z_n$	10 CROSS PRODUCTS $x_n z_{n+1}$	11 Twice Area Between Chord & Web	12 Effective Web Moment Constant	13 Summation of First Moments About C.G. Ax̄ + Az̄
									Rep. Pg.	Rep. Pg.	Rep. Pg.	⑤ - I	④ - I	Ax̄ ②⑤	Az̄ ②⑥	⑤ $_{n+1}$ ⑥ $_n$	⑤ $_n$ ⑥ $_{n+1}$	Rep. Pg.	⑨ - ⑩ + ⑪
								1											
								2											
								3											
								4											
								5											
								6											
								7											
								8											
								9											
								10											
								11											
								12											
								13											
								14											
								15											
								16											
								17											
								18											
								19											
								20											
								21											
								22											
								23											
								24											
								25											
								26											
								27											
								28											
								29											
								30											
								31											
								32											
								33											
								34											
								35											
								36											
								37											
								38											
								∑ checks				(a)	(a)	(b)	(b)			(c)	(b)

Values of $\bar{x}$ and $\bar{z}$ to two decimal places

$\bar{x} = \frac{\Sigma ⑤}{\Sigma ②} =$ _____ . $\bar{z} = \frac{\Sigma ⑥}{\Sigma ②} =$ _____ .

Arithmetical Checks:

(a) For columns ⑤ and ⑥, where n is the actual number of items,
$\Sigma ⑤ = \Sigma ③ - n\bar{x} =$ $\Sigma ⑥ = \Sigma ④ - n\bar{z} =$

(b) For columns ⑦, ⑧, & ⑬, $\Sigma ⑦ = 0$ (Approx.) ; $\Sigma ⑧ = 0$ (Approx.) $\Sigma ⑬ = 0$ (Approx.)

(c) For column ⑫, $\Sigma ⑫ = \Sigma ⑨ - \Sigma ⑩ + \Sigma ⑪$

Values of $\bar{I}$ to two decimal places

$I_x = \Sigma |⑧ ⑦| =$ $I_z = \Sigma |⑧ ⑧| =$ $I_{xz} = \Sigma |⑤ ⑦| =$
(Watch Signs) Check: $\Sigma I_{xz} = I_{xz} = \Sigma |⑬ ⑤ + ⑬ ⑨| =$

Values of bending constants:

To eight decimal places: $C_1 = \frac{1}{\left(\frac{I_x}{1000}\right)\left(\frac{I_z}{1000}\right) - \left(\frac{I_{xz}}{1000}\right)^2} =$

To three decimal places:

$C_2 = I_{xz} C_1 =$ $C_3 = I_z C_1 =$ $C_4 = I_x C_1 =$
(Same Sign as I_{xz})

14 Case	15 Condition	16 $10^4 M_x$	17 $10^4 M_z$	18 K_1	19 K_2
		Rep. Pg.	Rep. Pg.	C_3 ⑯ + C_4 ⑰	C_2 ⑯ + C_3 ⑰

of Tabular Form 1. The error introduced by assuming straight shear web units is in most cases very small and can be neglected.

If greater accuracy is desired, however, column 11 can be employed to any desired extent. For instance, if a correction is considered advisable for a portion of the leading edge of a D-tube (Fig. 13) it is only necessary to insert in column 11 the value $2A$ (*i.e.*, twice the area enclosed by the *straight* chord line of web 10 to 11 and the actual contour line of the web). Column 12 indicates the addition of the value, if any, in column 11, to the difference of columns 9 and 10 and gives twice the area enclosed, as shown in Fig. 13, between the straight lines from the c.g. to 10, from the c.g. to 11 and the contour line from 10 to 11. This method of correction has the advantage of confining direct measurements of area to very small portions of the cross-section, thereby reducing the labor and increasing accuracy.

The "effective web moment constants" obtained in column 12 are to be used later in determining the final shear flow.

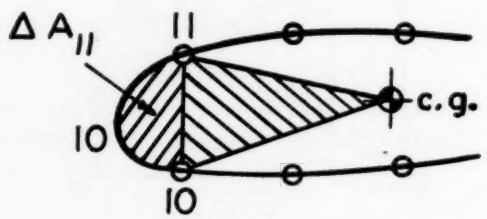


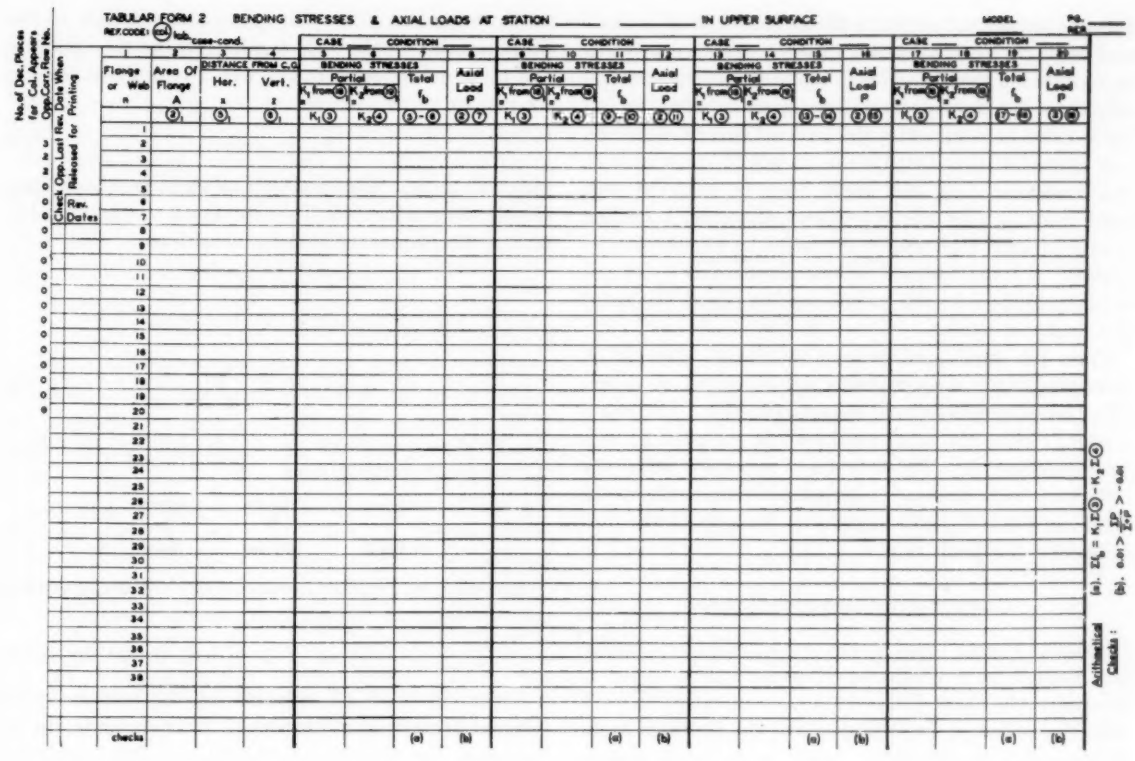
FIG. 13.

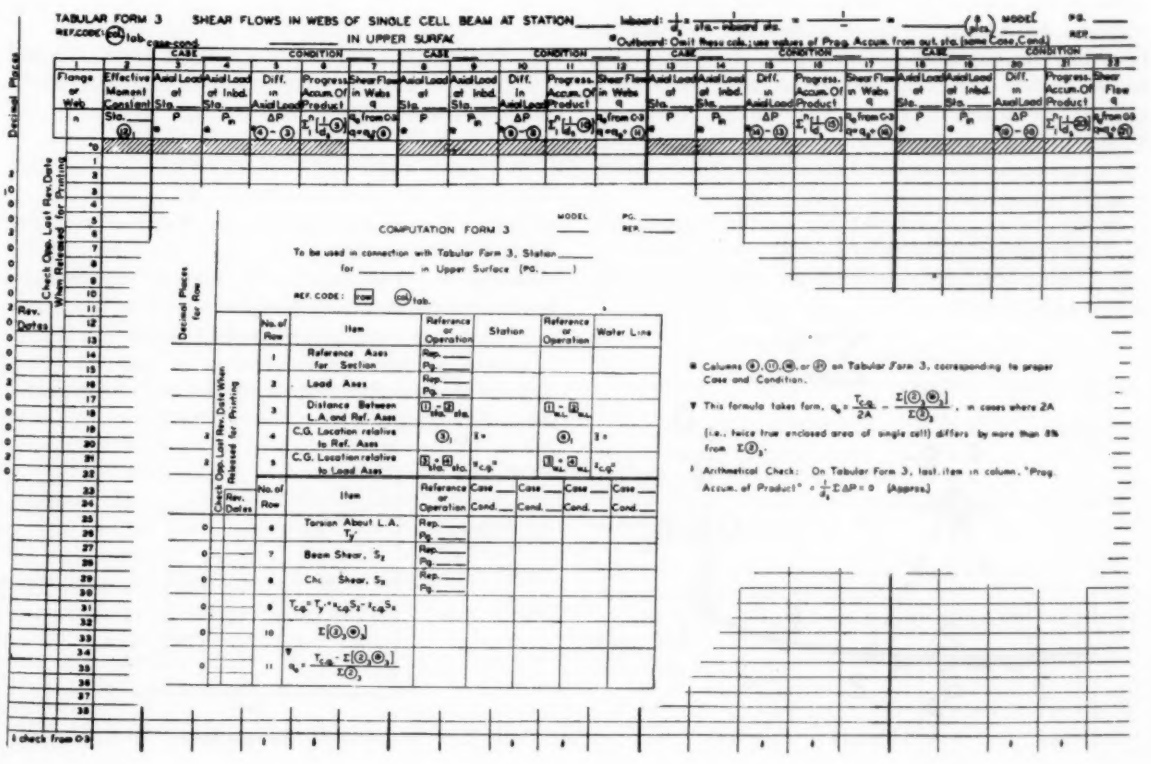
BENDING STRESSES

To avoid having to determine the principal axes (particularly bothersome in preliminary design work) the bending stresses are computed by the general formulas involving the bending moments about both reference axes (reference 6). These formulas have been converted to the sign conventions used in this paper. This involves the computation of constants K_1 and K_2 as indicated in columns 14 to 19 for different conditions of loading.

In determining the "stress constants" K_1 and K_2, the bending moments determined about the original reference axes are used directly. This means, in effect, that we have transferred these moments from the original reference axes (x', z') to the axes through the c.g. (x, z), a process which involves no corrections of any kind. (If principal axes had been employed, the bending moments would have had to be resolved into new components about those axes.) The total bending stresses are now obtained by filling in three more columns for each condition to be analyzed in Tabular Form 2 (such as columns 5, 6 and 7 for condition A). This completes the work required to convert the external bending moments into internal axial stresses. Preliminary trials of various flange material arrangements are facilitated by the small number of operations required.

We can now visualize the physical picture as follows. The external bending moments have been transferred to the c.g. and put into equilibrium by determining the


TABULAR FORM 2 BENDING STRESSES & AXIAL LOADS AT STATION _____ _____ IN UPPER SURFACE
MODEL
PG. _____
REF. _____
REF. CODE: row tab. case-cond.
No. of Dec. Places for Col. Appears
Opp. Corr. Row No.
Check Opp. Last Rev. Date When Released for Printing
Rev. Dates
CASE ____ CONDITION ____
Flange or Web n
Area Of Flange A
DISTANCE FROM C.G.
Hor. x
Vert. z
BENDING STRESSES
Partial
Total
Axial Load P
checks
Arithmetical Checks:
(a): $\Sigma f_b = K_1\Sigma(\text{col}) - K_2\Sigma(\text{col})$
(b): $\Sigma P = 0$


TABULAR FORM 3 SHEAR FLOWS IN WEBS OF SINGLE CELL BEAM AT STATION ____ Inboard: $\frac{1}{q_0}$ = Sta. – Inboard Sta. = ____ = ____ (Sta.) MODEL
PG. _____
REF. _____
REF. CODE: row tab. case-cond.
IN UPPER SURFAC
*Outboard: Omit Prog. Accum. use values of Prog. Accum. from next sta. (same Case, Cond.)
CASE ____ CONDITION ____
Flange or Web n
Effective Moment Constant
Sta. ____
Axial Load at Sta. ____ P
Axial Load at Inbd. Sta. ____ P_{in}
Diff. in Axial Load ΔP
Progress. Accum. Of Product $\Sigma\frac{\Delta P}{d_x}$
Shear Flow in Webs q
q_0 from C-3 $q = q_0 + (\text{col})$
Axial Load at Sta. ____ P
Axial Load at Inbd. Sta. ____ P_{in}
Diff. in Axial Load ΔP
Progress. Accum. Of Product $\Sigma\frac{\Delta P}{d_x}$
Shear Flow in Webs q
q_0 from C-3 $q = q_0 + (\text{col})$
Axial Load at Sta. ____ P
Axial Load at Inbd. Sta. ____ P_{in}
Diff. in Axial Load ΔP
Progress. Accum. Of Product $\Sigma\frac{\Delta P}{d_x}$
Shear Flow q
q_0 from C-3 $q = q_0 + (\text{col})$
Decimal Places for Row
Check Opp. Last Rev. Date When Released for Printing
Rev. Dates
COMPUTATION FORM 3
MODEL
PG. _____
REF. _____
To be used in connection with Tabular Form 3, Station ____
for ____ in Upper Surface (Pg. ____)
REF. CODE: row tab.
No. of Row
Item
Reference or Operation
Station
Water Line
1 Reference Axes for Section
2 Load Axes
3 Distance Between L.A. and Ref. Axes
4 C.G. Location relative to Ref. Axes
5 C.G. Location relative to Load Axes
Case ____ Cond. ____
6 Torsion About L.A. T_y'
7 Beam Shear, S_z
8 Chl. Shear, S_x
9 $T_{c.g.} = T_y' + x_{c.g.}S_z - z_{c.g.}S_x$
10 $\Sigma[(\text{col})(\text{col})_n]$
11 $q_0 = \frac{T_{c.g.} - \Sigma[(\text{col})(\text{col})_n]}{\Sigma(\text{col})_n}$
* Columns ⑥, ⑪, ⑯, or ㉑ on Tabular Form 3, corresponding to proper Case and Condition.
▼ This formula takes form, $q_0 = \frac{T_{c.g.}}{2A} - \frac{\Sigma[(\text{col})(\text{col})_n]}{\Sigma(\text{col})_n}$, in cases where 2A (i.e., twice true enclosed area of single cell) differs by more than 5% from $\Sigma(\text{col})_n$.
† Arithmetical Check: On Tabular Form 3, last item in column, "Prog. Accum. of Product" = $\frac{1}{d_x}\Sigma\Delta P = 0$ (Approx.)
‡ check from C-3

resisting axial stresses in the flange material, in accordance with the classical beam theory. There remain to be considered the shear loads along the original reference axes (x' and z') and the torque about axis y'. These must be resisted by the internal shear flow in the web material, the distribution of which must satisfy two conditions: (a) the shear flows in adjacent web units must satisfy the conditions determined by the change of axial load within the units, as previously described; (b) the internal torque about axis y (or y') must equal the external torque resulting from the shear and torsional loads.

Once the final arrangement of flange material is determined, the computations are extended to determine the shear flow around the cross-section. Column 8 (Form 2) gives the total axial load in each flange unit for condition A. These values, together with the "effective web moment constants," from column 12 (Tabular Form 1), will be used in Tabular Form 3 to determine the shear flow.

Computation of Shear Flow

Tabular Forms 1 and 2 now contain all the information needed at a particular cross-section in order to determine the shear flow. It is now necessary to work with two adjacent cross-sections of the beam in order to determine the change in "unit" axial loads between spanwise stations. This is done in Tabular Form 3, by entering the unit axial loads in columns 3 and 4 for condition A. Column 5 gives the change in axial load between the station in question and the next inboard station. In column 6 this is divided by the distance between stations and summed progressively, giving the shear flow increment due to change in axial loads between web 0 and any other web. The physical significance of this operation can be understood if we write the equation for the torsional moment of the shear flow in a given web unit.

In Fig. 14 we can pick web No. 2 as an example and find the torsional moment of a shear flow q_2 about the c.g. The general expression for the shear flow in any web may be written

$$q_n = q_0 + \sum_1^n \frac{\Delta P_n}{d_s} \tag{7}$$

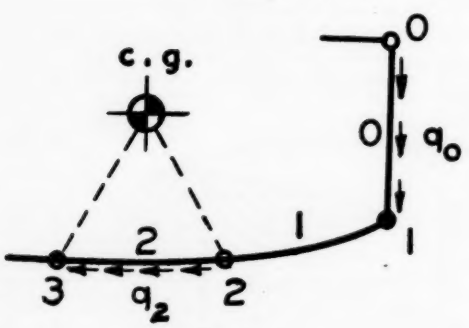


Fig. 14.

In this equation the subscript n refers either to the *flange* number or the web number, as the case may be. The torsional moment of the web No. 2 about the c.g. is equal to

$$T_{q_2} = q_2 C_2 \tag{8}$$

where C_2 is the "effective web moment constant" previously determined in Tabular Form 1 (column 12). Writing this equation in the general form:

$$T_{q_n} = q_n C_n$$

substituting for q_n from Eq. (7):

$$T_{q_2} = q_0 C_n + C_n \sum_1^n \frac{\Delta P_n}{d_s} \tag{9}$$

The total torque about the c.g. is obtained by adding the torques from each web:

$$T_{total} = q_0 \sum C_n + \sum \left(C_n \sum_1^n \frac{\Delta P_n}{d_s} \right) \tag{10}$$

Referring to Tabular Form 3, Eq. (10) can be written as

$$T_{total} = q_0 \sum \text{col. } 2 + \sum (\text{col. } 2)(\text{col. } 6) \tag{10a}$$

Assuming that all columns up to and including column 6 have been completed, we now have the information needed to determine q_0. The "external" torque about the c.g. is computed in Computation Form 3, from the known values of torque and shear loads at the reference axes:

$$T_{\text{c.g.}} = T_{y'} + x_{\text{c.g.}} S_z - z_{\text{c.g.}} S_x \tag{11}$$

Equating internal and external torques (Eqs. 10a and 11) and solving for q_0 we obtain the equation given in the Computation Form 3:

$$q_0 = \frac{T_{\text{c.g.}} - \sum ②⑥}{\sum ②} \tag{12}$$

The shear flow in each web is now obtained from Eq. (7) and the operation is carried out in column 7 for condition A. The shear stress may be obtained by dividing by the thickness of the web. (This last operation which has not been shown on Tabular Form 3, is, of course, subject to modification in case there is a division of shear flow between a flat and a corrugated sheet, for example.)

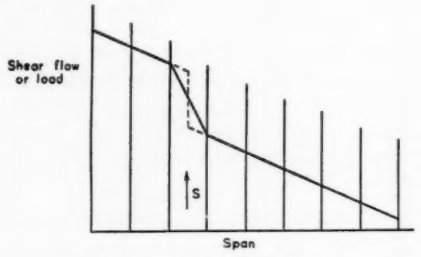


Fig. 15. Local effects.

The operations represented by columns 6 and 7 could be explained in a different manner by thinking of q_0 as the unknown *torsional* shear flow required to put the cross-section into equilibrium after it has been "cut" at web No. 0. Column 6 then represents the shear flows in the webs of the "cut" section, q_0 being assumed equal to zero for the "cut" web. The final shear flows are obtained by adding the "torsional" shear flow, q_0, to the "initial" shear flow, giving the same series of operations.

Modifications and Refinements

In applying the unit method to a wing, for example, the question of local loads may require special attention. One method of attack is to plot the shear flow, for a given web, against span and to modify this curve locally. Fig. 15 shows how this can be done.

The unit method will of course give the *average* shear flow over the unit length chosen. If a local load is introduced it is only necessary to extend the curves as shown by the dotted lines in Fig. 15. Actually, local conditions cannot be properly handled by the usual beam theory in any case, as the stress distribution will not conform to the classical beam theory near points where high local loads are applied.

The unit method of shear flow analysis may prove to be useful in connection with cut-outs and local discontinuities. For instance, it is possible to make arbitrary assumptions as to how the axial loads in the flange material depart from the beam theory around cut-outs. Once these assumptions have been made, the shear flow can be directly determined to correspond.

The use of a definite unit length over which the shear flow is averaged has a certain physical significance. It means, in effect, that local changes in the axial load distribution (such as caused by changes in gage, etc.) are assumed to be spread out over the unit length chosen. This is probably more nearly in accord with actual conditions than a more precise analysis which would utilize a very short unit length. In fact, further research may indicate what actual values of unit lengths should be used to simulate actual conditions for specific cases.

The vertical and horizontal shear load in a tapered beam is transmitted partially by the flanges and the balance by the web. Knowing the axial loads in the flanges and their respective angularity to the cross-section of the beam, the torsional moment about the c.g. due to axial loads in the flanges may be computed. The torsion (about the c.g.) to be transmitted by the web will be the difference between the torsion (about the c.g.) due to external loads and the torsion (about the c.g.) due to the flanges.

This refinement together with further refinements and extensions of the unit method will be taken up in a subsequent paper. The system has been worked out for the two-cell beam and can of course be further extended to the multi-cell case, if desired.

Appendix

Derivation of equation (5b) using Fig. 10

$$H_2 = \frac{M_2}{h_2} = \frac{S(d_1 + 2a)}{h_2} \qquad H_1 = \frac{M_1}{h_1} = \frac{Sd_1}{h_1}$$

$$q = \frac{H_2 - H_1}{2a} = \frac{S(d_1 + 2a)}{2ah_2} - \frac{Sd_1}{2ah_1}$$

$$= \frac{Sh_1(d_1 + 2a) - Sh_2d_1}{2ah_1h_2}$$

$$= \frac{S}{h_1h_2}\left[\frac{h_1d_1 + 2ah_1 - h_2d_1}{2a}\right]$$

$$= \frac{S}{h_1h_2}\left[\frac{h_1(d - a + 2a) - h_2(d - a)}{2a}\right]$$

$$= \frac{S}{h_1h_2}\left[\frac{h_1(d + a) - h_2(d - a)}{2a}\right]$$

$$= \frac{S}{h_1h_2}\left[\frac{(h - \Delta h)(d + a) - (h + \Delta h)(d - a)}{2a}\right]$$

$$= \frac{S}{h_1h_2}\left[\frac{hd + ha - d\Delta h - a\Delta h - hd + ah - d\Delta h + a\Delta h}{2a}\right]$$

$$= \frac{S}{h_1h_2}\left[\frac{2ah - 2d\Delta h}{2a}\right]$$

$$= \frac{S}{h_1h_2}\left[h - d\frac{\Delta h}{a}\right]$$

$$= \frac{1}{h_1h_2}\left[Sh - Sd(\tan\theta + \tan\alpha)\right]$$

Where $\frac{\Delta h}{a} = \tan\theta + \tan\alpha$

References

[1] Timoshenko, S., and MacCullough, Gleason H., *Elements of Strength of Materials*, 2nd edition, D. Van Nostrand Company, New York, 1940.

[2] Kuhn, Paul, *Some Elementary Principles of Shell Stress Analysis with Notes on the Use of the Shear Center*, N.A.C.A. Technical Note No. 691, 1939.

[3] Shanley, F. R., *Simplifying Stress Analysis*, Aviation, August, 1939.

[4] Hatcher, R. S., *Letter to the Editor*, Journal of the Aeronautical Sciences, Vol. 5, No. 9, July, 1938.

[5] Lockheed Stress Memorandum No. 16.

[6] Niles, Alfred S., and Newell, Joseph S., *Airplane Structures*, Volume I, 2nd edition, John Wiley & Sons, Inc., New York, 1938.

[7] Newell, Joseph S., *The Analysis of Leading Edge Wing Beams*, S.A.E. Journal, September, 1939.

[8] Cross, Hardy, *The Column Analogy*, University of Illinois, Bulletin No. 215.

Book Review

Graphical Treatment of Vibration, by C. H. POWELL; Bookcraft, New York, 1940; 288 pages, $7.50.

This book is in two sections, the first of which deals with the steady state vibration of a number of simple systems having a single vibratory degree of freedom. The second section, entitled "Solution of Vibration Damper Problems," deals with steady state vibration of simple systems having two vibratory degrees of freedom. The treatment is graphical except in the case of the analysis of the pendulum damper, which is set up by the use of La Grange's equation.

The promise implied by the end of the title, "Aircraft Engine Dampers," is not fulfilled since the book contains only a theoretical treatment of vibration dampers and no information on the application of dampers to engines.

No mention is made of transient vibration or of the important problem of vibration isolation. Systems which give rise to non-linear differential equations, and equations with variable coefficients, are not included; nor is any space devoted to systems having more than two vibratory degrees of freedom or having distributed mass and elasticity, although extension of the graphical method to the former case is suggested.

Dimensionless variables of various forms have been introduced but solution of the problems is always carried out in dimensional form first and then in one or more forms using dimensionless variables.

It may be concluded from the foregoing remarks that the problems treated in this book are of an elementary nature. Considerable confusion is caused by failure to state exactly the bodies upon which forces act. Consequently, the directions of forces are not always clear; for instance, on page 25 the equation $Q = hmA$ appears and on the same page the diagram is shown in which vectors representing the quantities Q and hmA are shown pointing in opposite directions. Further confusion is added because of the author's failure to use accepted notation and because of his use of familiar terms with meanings different from those ordinarily associated with them. For instance, m is used throughout for circular frequency in place of the accepted ω and I is used for mass and moment of inertia, which latter term the author shortens to "inertia." The term "linear vibrating system" is used to indicate the system in which the vibration is along a straight line instead of the usual use of the term indicating that only the first power of the variables and their derivatives appear in the differential equation representing the vibrating system. Further confusion is added by calling non-dimensional terms by the name of any quantity which appears in the numerator; for instance, on page 47 we are given the quantity $1/f = k/hm$, "Rigidity," while the inverse of this quantity is called "Frequency" or "Friction." The quantity g in the list of symbols is defined as Im/h, "a coefficient representing either Friction or Inertia (or Mass) in a system containing I, m and h. Reciprocal represent Friction."

The reader cannot help wondering if this apparent carelessness of expression extends to the many plots which are the chief virtue of the book. In these there are some apparent errors; for instance, on page 30, Fig. 11b shows dry (Coulomb) friction equal to zero when no slipping occurs; and in Fig. 125, page 153, the construction would indicate that

$$\frac{1}{\sqrt{1+c^2x^2}} + \frac{e+1}{\sqrt{1+c^2x^2}} = \frac{e}{\sqrt{1+c^2x^2}}.$$

The reviewer agrees with the author that a considerable insight into vibration problems can be had by a study of plots of the various functions which occur in the analysis of simple vibrating systems. The reviewer does not agree, however, that the author has justified the graphical approach to vibration problems, in view of the inferiority of this method in power and rigor as compared to the conventional treatment.

E. S. TAYLOR

Comment

Professor E. S. Taylor's wide recognition as an authority in the field of engine vibrations makes anyone hesitant to contradict his views on subjects in this field. In commenting upon a book, however, various viewpoints may be taken. An expert on the subject of vibrations, reading a book pertaining to this field, would very naturally differ in his reactions to the book from those of a person who is just beginning the study of vibration.

Professor Taylor states that the last part of the book title, "—and Aircraft Engine Dampers" is somewhat misleading. One would infer from the title that information is to be given on the specific application of dampers to aircraft engines. The word "aircraft" is hardly applicable.

As has been stated by Professor Taylor, the author has not been rigorous in his analytical treatment of the directions of the various vectors employed. The equation on page 25, for example, $Q = hmA$ is properly written as $Q = -hmA$ to denote that the restoring torque opposes the impressed torque. This equation would then conform to the vector diagram shown.

The notation used by the author is confusing since it does not conform to standard nomenclature. It is disconcerting to find the symbol g, which every technical reader would take as the acceleration due to the force of gravity, taken instead as a non-dimensional coefficient representing friction and/or inertia. It must be said that the author is consistent in his notation and the reader soon becomes accustomed to the system. Professor Taylor has covered the above features thoroughly in his review.

A certain number of errors generally appear in new books. The error in Fig. 125, page 153, mentioned by Professor Taylor is patently a typographical error. In the analytical treatment relating to this diagram, line CD is correctly considered as representing

$\frac{e-1}{\sqrt{1+c^2x^2}}$ instead of $\frac{e+1}{\sqrt{1+c^2x^2}}$ as shown. Another typographical error, not mentioned, occurs in the list of symbols in the introduction. The equation

$$m_* = \frac{k_2}{I_2} \text{ should be written as } m_*{}^2 = \frac{k_2}{I_2}$$

In the second section the following types of damper are discussed: the hysteresis damper, the Lanchester damper, the harmonic damper, and the pendulum damper. The pendulum damper does not lend itself as conveniently to the graphical treatment as do the others; consequently, the author finds it necessary to use La Grange's equations of motion in developing formulas for this damper. The curves for the various dampers have been carefully plotted and a study of these curves will prove beneficial to any reader. No specific applications of these dampers to aircraft engines are given, the entire treatment being theoretical in nature.

The book is limited in its scope, dealing only with systems having one or two degrees of freedom. The author does not treat such problems as systems with non-linear characteristics, systems having more than two degrees of freedom, transient vibrations, or the increasingly important problem of vibration isolation.

The book can be recommended to the reader who prefers a graphical treatment to an analytic one and who desires an introduction to vibration problems dealing with dampers.

Review

The book is in two sections. In the first section five simple but fundamental cases are treated and their vector diagrams are developed. The remainder of this section is devoted to the consideration of various combinations of the five fundamental cases. These combinations are "built up" in a very logical manner. Formulas are developed from the vector diagrams for the various amplitudes and angles of oscillation. The author, after developing his equations in dimensional form, makes use of non-dimensional coefficients which results in simplification of the formulas. In plotting the various quantities involved, these coefficients are very useful. It must be said that the notation used is confusing at first. Since the author is consistent the reader should have little difficulty.

Dr. Powell's treatment of the pendulum damper has been compared with Professor Taylor's treatment in his article "Crankshaft Torsional Vibration in Radial Aircraft Engines" appearing in the 1936 S.A.E. Transactions. The final results differ in form but can be reconciled since Dr. Powell considers the damper as a compound pendulum, whereas Professor Taylor considers the damper as a simple pendulum.

While most of the book's defects have been mentioned, nothing thus far has been said of its good features. I am submitting the following brief review of the book, having in mind that it is an introduction to the general subject of vibrations rather than a treatise written for the engineer who must consider all phases of the vibration problem. While the point of view taken by me will necessarily differ somewhat from that of Professor Taylor, there is no difference in the essentials.

MAURICE TUCKER

The Technical Information Service

of the

Institute of the Aeronautical Sciences

This service has experienced personnel under the supervision of trained aeronautical engineers to compile any information desired. The services range from listing specialized reference books to the preparation of exhaustive bibliographies, digesting of reports and general surveys of any aeronautical subject.

Some of the available services are:

Bibliographies on any aeronautical subject.

Reports on any aeronautical subject.

Digests of aeronautical books, papers, periodicals and references.

Translations in all languages.

Engineering investigations of special aeronautical subjects.

Biographies of individuals engaged in aeronautics.

Photostats of any aeronautical or general engineering material.

Microfilms made on special order.

Photographs made from the Institute's photographic collection.

Drawings and tracings made.

In addition to the services mentioned any commission which comes within the scope of the Service will be accepted.

Research work is charged at the usual library fee of $2.00 per hour. Special arrangements may be made for work requiring several weeks or months.

Translators are available for accurate transcriptions of all foreign language data. Translations are carefully edited by trained engineers at the standard rate of 1¢ per word. Minimum charge, $2.00.

Reproductions of any material in the Aeronautical Archives of the Institute may be ordered at standard photostat rates.

Institute Notes

Aircraft Steel Welding Contest

The American Welding Society has announced that a series of prizes has been established by the Summerill Tubing Company, Bridgeport, Pennsylvania, to be awarded by the Society at its annual meeting during October, 1941, for papers to advance the art of welding of aircraft steels including tubing and other steel parts for tubular assemblies. The conditions are as follows:

There will be awarded the following prizes provided that papers submitted are judged worthy of award: One $300 prize, one $200 prize and additional prizes totaling $100 to be distributed for such additional papers as the Committee of Awards may direct.

The papers may treat of any type of welding which is or can be used for the fabrication of structures or assemblies in the production of aircraft steels such as 1025, X4130, X4135, X4340 or similar steels. They may cover any phase of the problem whether concerning joint design, fabrication or laboratory investigations.

The contest is open to any resident of this country without restriction.

The Committee of Awards of the American Welding Society, with the addition of two men associated with the aircraft industry, will constitute the judges. The decision of the judges will be final.

Papers submitted in the contest may, at the discretion of the American Welding Society, be included in its next annual program or be published in The Welding Journal.

All papers must be typewritten, single spaced, written on one side only of blank white paper. Photographs, charts, graphs, etc., may either be attached directly to the copy or may be detached, in which case they should be clearly identified. Papers must be submitted not later than August 18, 1941, to the Aircraft Welding Contest, American Welding Society, 33 West 39th Street, New York, N. Y., from whom further information can be obtained if desired.

Personnel Changes

Members of the Institute are invited to use these columns to inform friends and associates of any professional changes they may make. All notices should be sent to the Editorial Secretary of the Journal.

Milton J. Thompson

Milton J. Thompson, Associate Professor in the Aeronautical Engineering Department at the University of Michigan, has accepted an offer from the University of Texas to take charge of its new aeronautical program. A graduate of Michigan in 1926, Prof. Thompson won the Daniel Guggenheim award in 1928 for two years study in aerodynamics under Prof. C. W. Witoszynski at Warsaw, Poland. In 1935 the book *Aerodynamic Theory*, edited by W. F. Durand, included *The Theory of Single Burbling* which Prof. Thompson wrote together with Prof. Witoszynski. His text on *Fluid Mechanics*, written in collaboration with R. A. Dodge, is well known.

George H. Tweney

George H. Tweney has recently joined the Faculty of the University of Detroit as Instructor in Aeronautics in the College of Engineering. He is an alumnus of the University, having obtained his Bachelor's Degree in Aeronautical Engineering in 1938. Prior to graduation, he served as undergraduate assistant in the University's aeronautical laboratories. In 1937 he was awarded first place in the Continental Aircraft Design Scholarship.

Gifts to the Institute

In addition to the gift to the Institute of prints and aeronautical pictures by Mrs. Bella C. Landauer which was announced at the Honors Night Dinner in January, there have been several additions to the art collection which the Aeronautical Archives is gathering. Charles L. Lawrance gave several medallions and Lester D. Gardner started a collection of aeronautical Christmas cards with a gift of nine volumes containing cards of aeronautical interest. A section containing calendars issued by aeronautical companies has been enlarged by gifts from printers.

So that the rare and more valuable books of the Institute could be carefully protected, Lester D. Gardner presented the Archives with two mahogany book cases.

The Paul Kollsman Library received two gifts of collections of books. Clarence D. Hanscom gave 197 technical aeronautical books, many of which are out of print and practically unprocurable. The value of these books was considered by the Council to warrant his election as a Benefactor Member. The other gift was made by W. A. M. Burden who presented over two hundred books. These were duplicates from his library and the gift makes them available for loaning.

Glenn D. Angle presented a collection of books on aircraft engines and a large collection of photographs which were used in his books on engines.

The Sawyer Electrical Manufacturing Company of Los Angeles presented a 10 hp. wind tunnel motor for exhibition.

Donald McNicol, Historian of the Telegraph Historical Society, added to the subject and subject files of the Archives by a gift of papers and pamphlets relating to early telegraphic experiments from balloons during the Civil War.

Personnel Opportunities

The Personnel Bureau serves individual members, as well as organizations seeking to employ aeronautical specialists. Any member or organization may have requirements listed without charge.

Wanted

Well established and progressive Ohio light aircraft manufacturer has openings for two experienced stress analysts and two experienced layout draftsmen. Frame structure experience mandatory, with sheet metal aircraft experience desirable. Submit full experience particulars, education, photograph, and salary expected in first letter. Address reply to Box 130, Institute of the Aeronautical Sciences.

Pennsylvania aircraft engine manufacturer urgently needs men of originality and well rounded experience for permanent positions, as follows: *Young Mechanical Engineer:* experienced in design and development of air-cooled engines by single-cylinder methods; *Layout Draftsman-Designer:* experienced in design of supercharged, air-cooled engines; *Draftsman-Checker:* having several years experience on aircraft engines; *Experienced Aircraft Engine Draftsman:* capable as Squad Leader for group of junior draftsmen. Address reply, giving complete experience record proving ability, stating salary expected, to Box 131, Institute of the Aeronautical Sciences.

Membership Roster, 1941–1942

Because of the many changes in location and activities of members of the Institute, publications of the Roster has been delayed. It will, therefore, be sent out with the May issue of the Journal.

Student Branches

Agricultural and Mechanical College of Texas. The Boeing School of Aeronautics film on "Weather Forecasting" was shown to 90 students at a meeting held on February 17. A. Lesher of the Aeronautical Engineering Department gave a brief talk on the work done in the development of the VS-300 Helicopter at a meeting held on March 3.

University of Alabama. Prof. Otto H. Lunde presented a paper on "The Development and Operation of the Autogiro" at a meeting held on February 24.

Boeing School of Aeronautics. Officers for the current term were elected as follows: H. H. Schunter, Chairman; Stanley Swanson, Vice-Chairman; and Charles Strausser, Secretary-Treasurer.

University of California. The following officers were elected at a meeting held on January 22: C. W. Harper, Chairman; Arthur Hughes, Vice-Chairman; and F. A. Wright, Secretary-Treasurer.

Carnegie Institute of Technology. At a meeting held on February 12, at which Chairman Marsh presided, Christian Wecker presented a paper on "Aviation Fuels."

University of Detroit. Richard C. Gazley of the Murray Corporation of America spoke on "New Worlds to Conquer in the Aeronautical Industry" at a meeting held on February 25. This was followed by the showing of the Institute film, "Streamline Visualization."

Iowa State College. The following officers were elected at the first meeting of this Student Branch on December 3: Prof. W. C. Nelson, Honorary Chairman; David Thurlow, Chairman; Warren Berkley, Vice-Chairman; Wilbur Glissman, Treasurer; and Charles Burkland, Secretary. The meeting was attended by 24 students who became charter members. Subsequent meetings were held on December 14, January 23 and on February 18. At the latter meeting, the Institute film, "Conquest of the Air," was shown.

University of Minnesota. The Fourth Wright Brothers Lecture on "Recent Fog Investigations" by Dr. Sverre Petterssen was presented by S. M. Serebreny at a meeting held on February 26. About 75 members were present.

Rensselaer Polytechnic Institute. Chairman Anderton presided at a meeting held on February 27 at which J. M. Carr gave a talk on "A High-Lift Wing" illustrated with motion pictures. Honorary Chairman Fairbanks gave a short talk on the life of Randolph Hall.

Changes in Membership

The following changes in the membership of the Institute have occurred since the publication of the previous list in the Journal.

Elected to MEMBER Grade

Banks, Charles Robinson, M.I.Ae.S.; Radio Development Engineer, American Airlines, Inc.

Burchett, Paul James, M.I.Ae.S.; Aero. Instructor, Pasadena Junior College.

Dudek, Edmund Frank, M.I.Ae.S.; Project Engineer, Vought-Sikorsky Aircraft, United Aircraft Corp.

Foa, Joseph Victor, Ae.E.; M.I.Ae.S.; Instructor, Univ. of Minnesota.

Gilbert, Roswell Ward, M.I.Ae.S.; Chief, Aircraft Engg. Div., Weston Electrical Instrument Corp.

Glad, Nicholas Constantin, M.I.Ae.S.; Design Engineer, Vought-Sikorsky Aircraft, United Aircraft Corp.

Hartman, Edwin Phelps, M.S.; M.I.Ae.S.; Assoc. Aero. Engineer, Ames Aero. Lab., N.A.C.A.

Heuver, Herbert Mannes, B.S. in Ae.E.; M.I.Ae.S.; Aero. Research Engineer, Aero. Lab., Univ. of Washington.

Knabenshue, Roy, M.I.Ae.S.; Aero. Assistant, National Park Service, Dept. of the Interior.

Malmsten, Lee Roy, Chief of Ground School, Embry-Riddle School of Aviation; Visiting Lecturer on Aero., Univ. of Miami.

McFarlane, Robert Morris, B.S.Ae.; M.I.Ae.S.; Sr. Stress Analyst, The Glenn L. Martin Co.

Pack, Mendel Nathan, M.S. in Ae.E.; M.I.Ae.S.; Chief of Structures Group, St. Louis Airplane Div., Curtiss-Wright Corp.

Petterssen, Sverre, Ph.D.; M.I.Ae.S.; Prof. of Meterology, Mass. Inst of Technology.

Pigman, George Leroy, B.S. in M.E.; M.I.Ae.S.; Asst. Engineer, C.A.A.

Resos, Eugene Peralta, B.S. in C.E.; M.I.Ae.S.; Stress Analyst, Vultee Aircraft, Inc.

Richter, Elvin Otto, M.I.Ae.S.; Wind Tunnel Operations Engineer, Lockheed Aircraft Corp.

Smith, George William, Jr., M.E.; M.I.Ae.S.; Chief Engineer, Barry & Wood.

Tjaarda, John, M.I.Ae.S.; Director of Design Research, Briggs Mfg. Co.

Sumner, Paul Alway, B.S. in M.E.; M.I.Ae.S.; Group Leader, Brewster Aeronautical Corp.

Ward, John Carlton, Jr., M.E.; M.I.Ae.S.; Pres., Fairchild Engine & Airplane Corp.

Watson, Harold Ernest, E.E.; M.I.Ae.S.; 1st Lt., U.S.A.; Engineer & Flying Officer, U.S. Army Air Corps.

Elected to Industrial Member Grade

Burke, Edward, M.I.Ae.S.; Vice-Pres., Aircraft Accessories Corp.

Kearney, Joseph Norman, M.I.Ae.S.; Pres., Defendaire Corporation.

Silsbee, Nathaniel Franklin, B.C.S.; M.I.Ae.S.; Comptroller, Beaver College; Aviation Writer and Lecturer.

Elected to Technical Member Grade

Bentley, Edward Patterson, Sc.D.; Research Associate, Aero. Dept., Mass. Inst. of Technology.

Chapman, Randall Noyes, B.Ae.E.; Head, Aero. Engg. Dept., Lawrence Inst. of Technology.

Chitwood, Harold Delmar, A.Ae.; Stress Analyst, Cessna Aircraft Corp.

Davies, Walter Moore, B.S. in E.E.; Aero. Draftsman, Curtiss Aeroplane Div., Curtiss-Wright Corp.

De Florez, Peter, B.S.; Installation Engineer, Ranger Aircraft Engines Div., Fairchild Engine & Airplane Corp.

Dillon, Frederick P., Sr. Research Engineer, Lockheed Aircraft Corp.

Douglas, Robert Bell, Engineer-Asst. Supt., Canadian Car & Foundry Co. Ltd., Canada.

Fromuth, Karl Ferdinand, B.S. in Ae.E.; Detail Design, Materiel Div., U.S. Army Air Corps, Wright Field.

Gordon, Haskell Robert, B.S. in Ae.E.; Asst. Aero. Engineer, U.S. Naval Aircraft Factory.

Gung, Simon Fenwick, Sr. Aircraft Draftsman, Engg. Dept., De Havilland Aircraft of Canada, Ltd., Canada.

Hamilton, Robert Bradley, B.S. in Ae.E.; Shop Liaison Engineer, Lockheed Aircraft Corp.

Hunter, David Ulrich, S.B. in M.E.; Sr. Stress Analyst, Wright Aeronautical Corp.

Ivanovic, Nicholas, B.S. in M.E.; Asst. Aero. Engineer, U.S. Naval Aircraft Factory.

Kahlke, Charles Edwin, Jr., Aircraft Engineer (Liaison), The Glenn L. Martin Co.

Lynt, Herbert Earl, Aircraft Instructor, Stewart Technical School.

Nicas, John Andrew, Checker, Noorduyn Aviation, Ltd., Canada.

Otis, Arthur Sinton, Ph.D.; Tech. Consultant, World Book Co.; Aviation Writer. (See A.M. of S.)

Pope, Alan Yatse, M.S. in Ae.E.; Asst. Prof. Aeronautics, Georgia School of Technology.

Stocking, Lowell Ellsworth, Armament Engineer, Douglas Aircraft Co.

TRANSFERRED TO ASSOCIATE FELLOW GRADE

Stanley A. Bell, Chief Engineer, Hughes Aircraft Co.; Fred N. Dickerman, Project Engineer, Vought-Sikorsky Aircraft, United Aircraft Corp.; Edward Harpoothian, Structural Designer, Douglas Aircraft Co.; Charles I. Lathrem, Chief Engineer, Propeller Div., Canadian Car and Foundry Co. Ltd.; William G. Ovens, Project Engineer, Wright Aeronautical Corp.; James D. Redding, Asst. Chief, Aircraft Airworthiness Section, C.A.A.

TRANSFERRED TO MEMBER GRADE

Howard H. Sargent. Chief Test Pilot, Pratt & Whitney Aircraft, United Aircraft Corp.; Alexander Satin, In charge Engineering & Drafting Depts., California Flyers School of Aeronautics.

NECROLOGY

HUTCHINSON INGHAM CONE

Rear Admiral Hutchinson Ingham Cone (U.S. Navy, Ret.), an Associate Fellow of the Institute, died at Orlando, Florida, on February 12, 1941. At the time of his death he was Chairman of the Board of the Directors, Moore & McCormack Co., steamship lines. He was in command of U.S. Naval Aviation Forces during the World War and had been an officer of the Daniel Guggenheim Fund for the Promotion of Aeronautics.

Admiral Cone was born in Brooklyn, N. Y., on April 26, 1871. He was educated in Florida schools, graduating from the Florida Agricultural College in 1889. He graduated from the U.S. Naval Academy in 1894, and served during the War with Spain under Dewey at Manila Bay. He commanded the torpedo boat flotilla on its voyage around Cape Horn in 1908, and continued as fleet engineer for the Atlantic Fleet on the trip around the world which was completed in 1909. He was then promoted from Lieutenant Commander to Rear Admiral and made Chief of the Navy Bureau of Steam Engineering.

He headed the Foreign Service of the U.S. Naval Aviation Forces, with headquarters in Dunkerque, France, from August, 1917 to October, 1918, when he was wounded on board the S.S. Leinster torpedoed in the Irish Sea.

After his retirement from the Navy in 1922 he became an executive of the Panama Steamship Company and later General Manager of the Emergency Fleet Corporation. He was a commissioner of the U.S. Shipping Board from 1928 to 1935.

Admiral Cone was a member of the Board of Directors of the Daniel Guggenheim Fund for the Promotion of Aeronautics, from 1925 to 1928, during which time he went abroad to make a study of foreign aviation for the Fund and served as Chairman of the Board, Vice-President and Treasurer.

His honors included the Distinguished Service Medal (U.S. Navy), Commander of the Order of the British Empire (Military Branch), Distinguished Service Order (British), and Officer of the Legion of Honor (French).

WILLIAM NATHAN MAY

William Nathan May, a Student Member of the Institute, died near Alexandria, Virginia, on January 27, 1941.

Mr. May was a senior at North Carolina State College and a native of Lenoir, North Carolina, where he was born on November 10, 1919. He was a graduate of Lenoir High School and entered State College in 1937. He was the Editor of "The Aeronaut," publication of the Student Branch of the Institute of the Aeronautical Sciences, and was an advanced student in the Civilian Pilot Training Program. He was a Sergeant in the Reserve Officers Training Corps during his junior year and a First Lieutenant during his senior year, having been recently assigned to Company "L," 3rd Battalion.

629

JOURNAL OF THE AERONAUTICAL SCIENCES

VOLUME 8 APRIL, 1941 NUMBER 6

Published in two Sections—Section One

THE JOHN CRERAR LIBRARY APR 21 1941

CONTENTS

Air-Cooled vs. Liquid-Cooled Aircraft . . . . . JOHN G. LEE 219

Letter to the Editor . . . . . FELIX NAGEL 229

Possibilities of the Two-Stroke Cycle for Small Aircraft Engines . . . . A. R. ROGOWSKI 230

On the Technique of Forecasting Low Ceilings and Fog . . . . . J. J. GEORGE 236

Flight Level Indicator . . . . . ROBERT W. KNIGHT 242

Unit Method of Beam Analysis . . . . . F. R. SHANLEY AND F. P. COZZONE 246

Book Review 256

Institute Notes 258

Published by the

INSTITUTE OF THE AERONAUTICAL SCIENCES INC.

JOURNAL OF THE AERONAUTICAL SCIENCES

VOL. 8 APRIL, 1941 NO. 6

Editor: J. C. HUNSAKER, *Massachusetts Institute of Technology*

PUBLISHED MONTHLY BY THE INSTITUTE OF THE AERONAUTICAL SCIENCES INC.
AT 20TH AND NORTHAMPTON STS., EASTON, PA.

The JOURNAL OF THE AERONAUTICAL SCIENCES is the official publication of the Institute of the Aeronautical Sciences Inc., and prints papers of scientific and technical interest to members and subscribers. In general, it provides an opportunity for specialists to publish articles disclosing new knowledge and new applications in the field of the aeronautical sciences. The Journal restricts itself to the aeronautical aspects of advances in science and engineering, and papers printed have either been submitted directly to the Journal or have been previously presented at a meeting of the Institute. News of the Institute is furnished members through the Journal.

The AERONAUTICAL REVIEW, which is published both as a section and separately as an edition of the Journal of the Aeronautical Sciences, keeps members and subscribers informed regarding the contents of new books and current periodicals in the field of aeronautics. It reviews new developments in the aircraft industry appearing in house organs and catalogues published by aircraft and engine manufacturers and other companies engaged in aeronautical activities.

Statements and opinions expressed in the Journal or the Aeronautical Review are to be understood as individual expressions of the authors, and not those of the Institute.

SUBSCRIPTION RATES

Journal of the Aeronautical Sciences Including the Aeronautical Review Section

United States and Possessions		Foreign Countries, Including Canada (American Currency Rates)	
1 Year	$10.00	1 Year	$12.50
Single Copies	1.00	Single Copies	1.25

Journal of the Aeronautical Sciences—Aeronautical Review Edition Only

United States and Possessions		Foreign Countries, Including Canada (American Currency Rates)	
1 Year	$2.50	1 Year	$3.00
Single Copies	.25	Single Copies	.30

Manuscripts for publication, proofs, and all correspondence should be addressed to the Editorial Secretary of the Journal.

Correspondence regarding subscriptions may be addressed to Publication Office, 20th and Northampton Sts., Easton, Pa., or to the Editorial Office of the JOURNAL OF THE AERONAUTICAL SCIENCES, 30 Rockefeller Plaza, Rockefeller Center, New York, New York. Cable address: INSAEROSCI.

Entered as Second Class Matter at the Post Office, Easton, Pa., May 1, 1937. Acceptance for mailing at a special rate of Postage provided for in the Act of August 24, 1912. Authorized April 29, 1937.

An Appreciation

TO CORPORATE MEMBERS OF THE INSTITUTE OF THE AERONAUTICAL SCIENCES

The publication of the Journal of the Aeronautical Sciences is made possible by Corporate Members who are manufacturers or operators of aircraft or producers of equipment, services or supplies used in aeronautics who contribute annual dues to the Institute.

The income from their dues enables the Institute to publish this section of the Journal without advertising and to devote its pages exclusively to editorial material. The Members of the Institute appreciate the cooperation of Corporate Members in providing funds for assisting the work of the Institute and for the publication of the Journal.

AERO INSURANCE UNDERWRITERS
59 John Street, New York, N. Y.

AIRCRAFT RADIO CORPORATION
Boonton, New Jersey

ALUMINUM COMPANY OF AMERICA
Gulf Building, Pittsburgh, Pennsylvania

AMERICAN AIRLINES
Chicago Municipal Airport, Chicago, Illinois

THE B. G. CORPORATION
136 W. 52nd Street, New York, N. Y.

BENDIX AVIATION CORPORATION
30 Rockefeller Plaza, New York, N. Y.

EXPORT DIVISION
New York, N. Y.

BENDIX PRODUCTS DIVISION
South Bend, Indiana

BENDIX STROMBERG CARBURETOR DEPT.

BENDIX AIRCRAFT LANDING GEAR DEPT.

BENDIX AVIATION CORPORATION
30 Rockefeller Plaza, New York, N. Y.

ECLIPSE AVIATION DIVISION
Bendix, New Jersey

JULIEN P. FRIEZ & SONS DIVISION
Baltimore, Maryland

BENDIX RADIO CORPORATION
Baltimore, Maryland Chicago, Illinois

PIONEER INSTRUMENT DIVISION
Bendix, New Jersey

SCINTILLA MAGNETO COMPANY
Sidney, New York

BOEING AIRCRAFT COMPANY
Seattle, Washington

BOEING SCHOOL OF AERONAUTICS
Oakland Municipal Airport, Oakland, California

EDWARD G. BUDD MANUFACTURING COMPANY
Philadelphia, Pennsylvania

CARNEGIE-ILLINOIS STEEL CORPORATION
UNITED STATES STEEL CORPORATION SUBSIDIARY
Carnegie Building, Pittsburgh, Pennsylvania

THE CLEVELAND PNEUMATIC TOOL COMPANY
AUTOMOTIVE-AIRCRAFT DIVISION
3734 East 78th Street, Cleveland, Ohio

AEROL STRUTS
ASK THE PILOTS WHO LAND ON THEM

Consolidated Aircraft Corporation
Lindbergh Field, San Diego, California

Curtiss-Wright Corporation
30 Rockefeller Plaza, New York, N. Y.

St. Louis Airplane Division
Robertson, Missouri

Curtiss Aeroplane Division
Buffalo, New York

Curtiss-Wright Propeller Division
Clifton, New Jersey

Wright Aeronautical Corporation
Paterson, New Jersey

Douglas Aircraft Company
Santa Monica, California

El Segundo Division
El Segundo, California

Edo Aircraft Corporation
College Point, Long Island, New York

Ethyl Gasoline Corporation
135 East 42nd Street, New York, N. Y.

Fairchild Aviation Corporation
30 Rockefeller Plaza, New York, N. Y.

Fairchild Aerial Camera Corporation
Jamaica, Long Island, New York

Fairchild Aerial Surveys
Los Angeles, California

Fairchild Engine & Airplane Corporation
30 Rockefeller Plaza, New York, N. Y.

Fairchild Aircraft Corporation
Hagerstown, Maryland

Ranger Engineering Corporation
Farmingdale, Long Island, New York

General Tire & Rubber Company
Akron, Ohio

The B. F. Goodrich Company
Akron, Ohio

The Goodyear Tire & Rubber Company
Akron, Ohio

Grumman Aircraft Engineering Corp.
Bethpage, L. I., New York

Gulf Oil Corporation
Gulf Building, Pittsburgh, Pa.

W. & L. E. Gurley
Troy, N. Y.

The International Nickel Company
67 Wall Street, New York, N. Y.

Kollsman Instrument Division
of Square D Company
80-08 45th Avenue, Elmhurst, L. I., N. Y.

KOLLSMAN
FOR
PRECISION

LOCKHEED AIRCRAFT CORPORATION
Burbank, California

THE GLENN L. MARTIN COMPANY
Baltimore, Maryland

NORMA-HOFFMANN BEARINGS CORPORATION
Stamford, Connecticut

NORTHWEST AIRLINES, INC.
St. Paul Municipal Airport, St. Paul, Minnesota

PAN AMERICAN AIRWAYS
135 East 42nd Street, New York City

PUMP ENGINEERING SERVICE CORPORATION
DIVISION OF BORG-WARNER CORPORATION
12910 Taft Avenue, Cleveland, Ohio

THE PURE OIL COMPANY
Pure Oil Building, 35 East Wacker Drive, Chicago, Illinois

RCA MANUFACTURING COMPANY
Camden, New Jersey

SHELL OIL COMPANY
50 West 50th Street, New York, N. Y.
Shell Building, St. Louis, Missouri
Shell Building, San Francisco, California

SOCONY-VACUUM OIL COMPANY
26 Broadway, New York, N. Y.

SPERRY GYROSCOPE COMPANY
Manhattan Bridge Plaza, Brooklyn, New York

STANAVO SPECIFICATION BOARD, INC.
225 Bush Street, San Francisco, California

STANDARD OIL COMPANY OF CALIFORNIA
225 Bush Street, San Francisco, California

STANDARD OIL COMPANY (INDIANA)
910 S. Michigan Ave., Chicago, Ill.

STANDARD OIL COMPANY OF NEW JERSEY
26 Broadway, New York, N. Y.

SUMMERILL TUBING COMPANY
Bridgeport, Montg. Co., Pennsylvania

THE TEXAS COMPANY
135 East 42nd Street, New York, N. Y.

THOMPSON PRODUCTS, INC.
Cleveland, Ohio

TRANSCONTINENTAL & WESTERN AIR
Municipal Airport, Kansas City, Mo.

UNITED AIRCRAFT CORPORATION
East Hartford, Connecticut

HAMILTON STANDARD PROPELLERS
East Hartford, Connecticut

PRATT & WHITNEY AIRCRAFT
East Hartford, Connecticut

VOUGHT-SIKORSKY AIRCRAFT DIVISION
Stratford, Connecticut

UNITED AIR LINES TRANSPORT CORPORATION
5959 S. Cicero Avenue, Chicago, Illinois

UNITED STATES AVIATION UNDERWRITERS
80 John Street, New York, N. Y.

WESTINGHOUSE ELECTRIC AND MFG. COMPANY
East Pittsburgh, Pennsylvania

WYMAN-GORDON COMPANY
Worcester, Massachusetts

PRODUCTS AND SERVICES OF CORPORATE MEMBERS OF THE INSTITUTE OF THE AERONAUTICAL SCIENCES

The companies listed below according to the types of products or services which they supply are Corporate Members contributing directly to the expenses of the Institute's activities and the publication of the Journal. Members of the Institute can show their appreciation by recommending the products and services of Corporate Members whenever possible.

AIRCRAFT
Boeing Aircraft Company
Edward G. Budd Manufacturing Company
Consolidated Aircraft Corporation
Curtiss Aeroplane Division, Curtiss-Wright Corporation
Douglas Aircraft Company
Fairchild Aircraft Corporation
Grumman Aircraft Engineering Corporation
Lockheed Aircraft Corporation
The Glenn L. Martin Company
St. Louis Airplane Division, Curtiss-Wright Corporation
Vought-Sikorsky Aircraft Division, United Aircraft Corporation

AIR TRANSPORTATION
American Airlines
Northwest Airlines
Pan American Airways
Transcontinental & Western Air
United Air Lines Transport Corporation

AIRPORT TRAFFIC CONTROL EQUIPMENT
Bendix Radio Corporation
Julien P. Friez & Sons Division, Bendix Aviation Corp.
RCA Manufacturing Company

BEARINGS (BALL, ROLLER, THRUST)
Norma-Hoffmann Bearings Corp.

CAMERAS
Fairchild Aerial Camera Corporation

CARBURETORS
Bendix Products Division, Bendix Aviation Corp. (Stromberg Carburetor Department)

COLLECTOR RINGS, COWLS, STREAMLINES
Curtiss Aeroplane Division, Curtiss-Wright Corporation

CONSTRUCTION MATERIALS

Aluminum Company of America
Edward G. Budd Manufacturing Company
Carnegie-Illinois Steel Corporation
Eclipse Aviation Division, Bendix Aviation Corp.
The International Nickel Company
Summerill Tubing Company

DE-ICER EQUIPMENT

Eclipse Aviation Division, Bendix Aviation Corp.
The B. F. Goodrich Company

DROP FORGINGS

Wyman-Gordon Company

EDUCATION

Boeing School of Aeronautics

ENGINE PARTS & ACCESSORIES

Bendix Products Division, Bendix Aviation Corp. (Stromberg Carburetor Department)
Eclipse Aviation Division, Bendix Aviation Corp.
Scintilla Magneto Company
Thompson Products, Inc.

ENGINES

Pratt & Whitney Aircraft Division, United Aircraft Corporation
Ranger Engineering Corporation
Wright Aeronautical Corporation

FLOATS, SKIS

Edo Aircraft Corporation

FUEL & HYDRAULIC SYSTEMS

Eclipse Aviation Division, Bendix Aviation Corp.
Pump Engineering Service Corporation

FUELS & OILS

Ethyl Gasoline Corporation
Gulf Oil Corporation
The Pure Oil Company
Shell Oil Company
Socony-Vacuum Oil Company
Stanavo Specification Board, Inc.
Standard Oil Company of California
Standard Oil Company (Indiana)
Standard Oil Company of New Jersey
The Texas Company

GEAR BOXES (ACCESSORY DRIVE)

Pump Engineering Service Corporation

GENERATORS

Eclipse Aviation Division, Bendix Aviation Corp.
Westinghouse Electric and Manufacturing Company

HYDRAULIC MEASURING INSTRUMENTS

W. & L. E. Gurley

INSTRUMENTS & CONTROL UNITS

Eclipse Aviation Division, Bendix Aviation Corp.
Fairchild Aerial Camera Corporation
Julien P. Friez & Sons Division, Bendix Aviation Corp.
Kollsman Instrument Division of Square D Company
Pioneer Instrument Company
Sperry Gyroscope Company
Westinghouse Electric and Manufacturing Company

INSURANCE

Aero Insurance Underwriters
United States Aviation Underwriters

LANDING GEAR & SHOCK STRUTS

Bendix Products Division, Bendix Aviation Corp. (Aircraft Landing Gear Department)
The Cleveland Pneumatic Tool Company
Edo Aircraft Corporation
Thompson Products, Inc.

MAGNETOS

Eclipse Aviation Division, Bendix Aviation Corp.
Scintilla Magneto Company

NAVIGATION INSTRUMENTS

W. & L. E. Gurley

NICKEL & NICKEL ALLOYS

The International Nickel Company

PILOT SEATS

Bendix Products Division, Bendix Aviation Corp.
(Aircraft Landing Gear Department)

PNEUMATIC TOOLS FOR AIRCRAFT

The Cleveland Pneumatic Tool Company

PROPELLERS & PROPELLER PARTS

Curtiss Propeller Division, Curtiss-Wright Corporation
Eclipse Aviation Division, Bendix Aviation Corp.
Hamilton Standard Propellers Division, United Aircraft Corporation

PUBLIC ADDRESS SYSTEMS

RCA Manufacturing Company

PUMPS (AIR, FUEL, OIL, VACUUM HYDRAULIC)

Eclipse Aviation Division, Bendix Aviation Corp.
Pump Engineering Service Corporation
Thompson Products, Inc.

RADIO APPARATUS FOR AIRCRAFT & AIRPORTS

Aircraft Radio Corporation
Bendix Radio Corporation
Eclipse Aviation Division, Bendix Aviation Corp.
Fairchild Aerial Camera Corporation
RCA Manufacturing Company
Sperry Gyroscope Company
Westinghouse Electric and Manufacturing Company

REMOTE CONTROL & RECORDING DEVICES

Bendix Radio Corporation
Eclipse Aviation Division, Bendix Aviation Corp.
Julien P. Friez & Sons Division, Bendix Aviation Corp.
Pioneer Instrument Company
RCA Manufacturing Company
Westinghouse Electric and Manufacturing Company

RUBBER, SYNTHETICS

The B. F. Goodrich Company

SPARK PLUGS

The B. G. Corporation
Scintilla Magneto Company

STAINLESS STEEL

Carnegie-Illinois Steel Corporation

STARTERS

Eclipse Aviation Division, Bendix Aviation Corp.

SURVEYING INSTRUMENTS

W. & L. E. Gurley

SURVEYS (AERIAL)

Fairchild Aerial Camera Corporation

SWITCHES

Eclipse Aviation Division, Bendix Aviation Corp.
Scintilla Magneto Company
Westinghouse Electric and Manufacturing Company

TIRES & TUBES

General Tire & Rubber Company
The B. F. Goodrich Company
The Goodyear Tire & Rubber Company

TUBING

Aluminum Company of America
Carnegie-Illinois Steel Corporation
Eclipse Aviation Division, Bendix Aviation Corp.
The International Nickel Company
Summerill Tubing Company

VALVES & ACTUATING CYLINDERS

Pump Engineering Service Corporation
Thompson Products, Inc.

VIBRATION DAMPERS

General Tire & Rubber Company

WEATHER INSTRUMENTS

Julien P. Friez & Sons Division, Bendix Aviation Corp.

STANDARD PRECISION WEIGHTS AND MEASURES

W. & L. E. Gurley

WHEELS & BRAKES

Bendix Products Division, Bendix Aviation Corp.
(Aircraft Landing Gear Department)
The Goodyear Tire & Rubber Company

INSTITUTE of the AERONAUTICAL SCIENCES

President
FRANK W. CALDWELL

Vice Presidents
W. A. M. BURDEN
CHARLES H. COLVIN
HALL L. HIBBARD
PHILIP G. JOHNSON

Executive Vice-President
LESTER D. GARDNER

Past Presidents
J. C. HUNSAKER
CHARLES L. LAWRANCE
D. W. DOUGLAS
GLENN L. MARTIN
CLARK B. MILLIKAN
T. P. WRIGHT
GEORGE W. LEWIS
JAMES H. DOOLITTLE

Secretary
C. E. SINCLAIR

Treasurer
GROVER LOENING

Assistant Treasurer
GEORGE R. FORMAN

Controller
JOSEPH J. MAITAN

COUNCIL

RALPH S. DAMON
LUIS DE FLOREZ
RICHARD GOLDSMITH
LEROY R. GRUMMAN
RAYMOND HASKELL
W. WALLACE KELLETT
BRUCE G. LEIGHTON
ALBERT I. LODWICK
R. D. MACCART
CHARLES MARCUS
ARTHUR NUTT
ELMER A. SPERRY, JR

ADVISORY BOARD

JOHN D. AKERMAN
LAWRENCE D. BELL
VINCENT BENDIX
GEORGE H. BRETT
LYMAN J. BRIGGS
JACK FRYE
S. PAUL JOHNSTON
J. H. KINDELBERGER
F. K. KIRSTEN
A. L. KLEIN
A. KLEMIN
PAUL KOLLSMAN
HAROLD F. PITCAIRN
A. E. RAYMOND
F. W. REICHELDERFER
M. J. THOMPSON
J. H. TOWERS
J. T. TRIPPE
A. D. TUTTLE
E. P. WARNER

HONORARY MEMBERS

H. H. ARNOLD
HARLLEE BRANCH
GEORGE H. BRETT
GRIFFITH BREWER
LYMAN J. BRIGGS
DELOS C. EMMONS
HARRY F. GUGGENHEIM
ROBERT H. HINCKLEY
GEORGE W. LEWIS
GEORGE B. PEGRAM
F. W. REICHELDERFER
J. H. TOWERS

HONORARY FELLOWS

J. S. AMES
A. CAQUOT
G. A. CROCCO
D. W. DOUGLAS
HUGH L. DRYDEN
W. F. DURAND
HUGO ECKENER
A. H. R. FEDDEN
J. C. HUNSAKER
B. MELVILL JONES
TH. VON KÁRMÁN
GEORGE W. LEWIS
GLENN L. MARTIN
GEORGE J. MEAD
LUDWIG PRANDTL
D. R. PYE
G. I. TAYLOR
ORVILLE WRIGHT

BENEFACTOR MEMBERS

E. E. ALDRIN
HART O. BERG
E. L. CORD
D. W. DOUGLAS
SHERMAN M. FAIRCHILD
C. R. FAIREY
R. H. FLEET
C. D. HANSCOM
PAUL KOLLSMAN
BELLA C. LANDAUER
GROVER LOENING
EDMUND C. LYNCH (Deceased)
F. W. MAGIN
GLENN L. MARTIN
GEO. J. MEAD
S. A. REED (Deceased)
LAURANCE S. ROCKEFELLER

AMERICAN FELLOWS

EDWIN E. ALDRIN
EDMUND T. ALLEN
KARL ARNSTEIN
J. L. ATWOOD
RALPH S. BARNABY
P. R. BASSETT
REX B. BEISEL
G. M. BELLANCA
DON R. BERLIN
HENRY A. BERLINER
B. C. BOULTON
LYMAN J. BRIGGS
W. G. BROMBACHER
CHARLES P. BURGESS
FRANK W. CALDWELL
CHARLES H. CHATFIELD
ALLAN CHILTON
V. E. CLARK
CHARLES H. COLVIN
KARL T. COMPTON
A. V. DE FOREST
SMITH J. DE FRANCE
J. P. DEN HARTOG
WALTER S. DIEHL
J. H. DOOLITTLE
C. S. DRAPER
C. L. EGTVEDT
GARLAND FULTON
LESTER D. GARDNER
RICHARD C. GAZLEY
MELVIN N. GOUGH
LEROY R. GRUMMAN
ALBERT F. HEGENBERGER
S. D. HERON
HALL L. HIBBARD
L. S. HOBBS
WILLIAM HOVGAARD
ROBERT INSLEY
EASTMAN N. JACOBS
PAUL H. KEMMER
J. H. KIMBALL
J. H. KINDELBERGER
ALEXANDER KLEMIN
W. B. KLEMPERER
PAUL KOLLSMAN
OTTO C. KOPPEN
S. M. KRAUS
I. MACHLIN LADDON
E. S. LAND
CHARLES L. LAWRANCE
E. P. LESLEY
G. W. LEWIS
CHARLES A. LINDBERGH
WILLIAM LITTLEWOOD
GROVER LOENING
R. D. MACCART
WM. H. MCAVOY
C. J. MCCARTHY
LESSITER C. MILBURN
CLARK B. MILLIKAN
ROBERT J. MINSHALL
MAX M. MUNK
JOSEPH S. NEWELL
ALFRED S. NILES
JOHN K. NORTHROP
ARTHUR NUTT
ROBERT R. OSBORN
W. BAILEY OSWALD
GEORGE A. PAGE, JR.
F. W. PAWLOWSKI
A. A. PRIESTER
ARTHUR E. RAYMOND
F. W. REICHELDERFER
ELLIOTT G. REID
H. J. E. REID
HOLDEN C. RICHARDSON
C. E. ROSENDAHL
C.-G. ROSSBY
MAC SHORT
IGOR ALEXIS SIKORSKY
IGOR I. SIKORSKY
ELMER A. SPERRY, JR.
WILLIAM B. STOUT
C. FAYETTE TAYLOR
EDWARD S. TAYLOR
PHILIP B. TAYLOR
TH. THEODORSEN
S. TIMOSHENKO
STARR TRUSCOTT
L. B. TUCKERMAN
RALPH H. UPSON
EDWARD P. WARNER
FRED E. WEICK
R. D. WEYERBACHER
ANDREW V. WILLGOOS
DONALD H. WOOD
T. P. WRIGHT
JOHN E. YOUNGER
ALBERT F. ZAHM
STEPHEN J. ZAND

INSTITUTE *of the* AERONAUTICAL SCIENCES

AIMS

The Institute of the Aeronautical Sciences is organized to promote the application of science in the development of aircraft. By grading members according to their qualifications, recognition is given to outstanding professional work and an incentive is provided younger members for the investigation of new aeronautical problems. The presentation and discussion of technical papers at Institute meetings afford members opportunities to hear and meet the leaders in the various aeronautical sciences. Through the Journal of the Aeronautical Sciences, members are provided with an opportunity to publish and to read of new knowledge and new applications in the field of aeronautical engineering. The continuous improvement of aircraft depends on scientific research and engineering experiment applied to design. The Institute desires to bring together all specialists contributing to aeronautical progress and, for this purpose, is keeping the dues nominal in amount, in view of the fact that practically every specialist already belongs to his own special professional society. The dues from members do not provide funds enough both for publication of the Journal and for the Institute's other activities, the difference being made up from funds contributed by benefactors and by the aeronautical industry.

The headquarters of the Institute provides members when in New York with a convenient and attractive meeting place.

DUES

The entrance fee for all members is five dollars. The annual dues are: Fellows, $20.00; Associate Fellows, $15.00; MEMBERS, $12.50; Industrial Members, $12.50; Technical Members, $7.50.

APPLICATION for MEMBERSHIP

Application for membership forms may be secured from any member of the Institute or from the Secretary. An Applicant is required to furnish the names of several members acquainted with his professional work. The Admissions Committee recommends the grade of membership to the Council which, at its discretion, may extend to the Applicant an invitation to join the Institute.

INSTITUTE OF THE AERONAUTICAL SCIENCES

1505 RCA BUILDING WEST

30 ROCKEFELLER PLAZA

NEW YORK